

TRIGONOMETRY,  
WITH THE  
DOCTRINE of the *Sphere*;  
PLAIN DIALING  
AND  
SPHERIQUE GEOMETRY,  
OR  
PROJECTION of the *Sphere in Plano*:

To which is subjoyn'd

AN APPENDIX

Applying PLAIN TRIGONOMETRY to the taking of HEIGHTS and DISTANCES, and to Plain and Mercator's SAILING.

---

The Second EDITION, more full and correct than the former.

---

To the whole is added a short *Treatise* of SECTIONS of PLANS, and an *Introduction* to ARCHITECTURE.

---

*Quicquid agunt homines* ———  
——— *nostri est farrago libelli.*

---

EDINBURGH,

Printed by *William Brown* and *John Mosman*, the Assigns of *James Watson* deceased, His Majesty's Printers. M. DCC. XXIV.





241  
4

i



# THE PREFACE.

READER,

**I** Shall not trouble you with a long  
Detail of the Motives that indu-  
ced me first of all to publish this  
small Treatise of Trigonometry, and  
the Doctrine of the Sphere. In the  
Preface to the first Edition I gave a  
Hint of them, and I'm not very guilty  
of vain Repetitions at any Time. The  
Reason why I give this new Edition  
is, it was out of Print, and much wan-  
ted. I have subjoyned a small Tract  
of Sections of Plans, in order to the  
more easy and commodious Laying out  
of Ground; it being the Happiness of  
† our

## ii The PREFACE.

*our Country at this Time, that our Nobility and Gentlemen of Fortunes find it at once an advantageous Exercise, and agreeable Amusement to improve their Fortunes. The Doctrine it self Mr. Leyburn lays down dogmatically in his complete Surveyor, I, for my Part, have only added the Demonstrations.*

*The short Treatise of Architecture I published (don't laugh, Reader) at the earnest Intreaty of Friends. You'll perhaps say, this is a trite and common Excuse. I own it is; but I declare solemnly it is true: And I found it easier, and more gentlemanny too (if the Word may be admitted) to tell this sincere out of fashioned Truth, than to invent a new Apology and Lie.*

*Indeed a Treatise on this Subject is much wanted at present. Our Mechanics labour under a grand Defect that Way; and how far it may be of Use to our young Nobility and Gentry to read something on that Subject before they go to their Travels, is best known to those who, without any previous Knowledge of this Art, have travelled, and,*  
at



## The PREFACE. iii

*at their Return home, could only tell in general that they saw a great many beautiful Buildings abroad, without being capable of pointing out one particular Beauty in any one of them; if they should pretend to give a Description of any one, it would be done in such Terms as would make an Artist's Head ache. This I mention as their Misfortune, not their Fault, there being, in this Place, no Way of attaining to any Knowledge in this Art, but by reading vast and many Volumes, which some of our great Men have done with such surprising Success, that their Knowledge in this, and other Sciences, has added a fresh Lustre to the Grandeur of their Birth.*

*I have here laid down the first Elements of Architecture, without troubling my Reader with long Encomiums on the Beauties of the several Parts, or a tedious Story of the Rise and Advancement of it. The first of these being altogether needless, the Beauties of the several Parts being obvious to the Eye, and the latter being noways essential to the Subject in Hand, I thought*  
*it*

## iv The PREFACE.

*it would be more convenient to tell it by Word of Mouth in the Course of my Lectures on this Treatise, than to swell the Treatise it self to a large Volume with it.*

*In the Spherique Trigonometry I have demonstrated a short and easy Method of resolving that perplext Problem of finding the Angles of a spherique Triangle from the Three Sides given. This Problem I find demonstrated no where but in a small Piece of Trigonometry written by our noble and learned Countryman, the Lord Napier, Baron of Merchistoun, where he takes some Things for granted, for which he refers his Reader to Regiomontanus; these I lay down beforehand as Lemmata. To make this Course of Trigonometry as complete as possible, I have added to it that noble Lord's Catholicism. If what I here advance shall please thee, Reader, you may expect after this to hear more from,*

*Your humble Servant,*

JO. WILSON.

# Explication of CHARACTERS.

Equal.

Greater.

Lesser.

*Plus*, more, or to be added.

*Minus*, less, or to be subtracted.

The Difference where it is uncertain  
which Quantity is the greater.

Multiplication or Rectangle.

The Quantity above to be divided by  
that below the Quote or Quotient.

Geometrical Proportion discrete.

—————continued.

Arithmetical Proportion discrete.

—————continued.

Degree.

Minute, both in Sphericks and Ar-  
chitecture.

Triangle.

Angle,  $\angle \angle$  Two Angles, &c.

Angle opposite to a given Side.

Angle adjacent to a given Side.

Angle intercepted betwixt Two Sides.

*Latus*, Side.

Side opposite to a given Angle, &c.  
as in Angles.

Perpendicular.

Parallel.

Sine.

Co-Sine.

Tangent.

Co-Tangent.

Module.

Signifies

ER-



## ERRATA.

- P**Age 210. Line 21. for **AE** read **AF**.  
P. 239. under L. 3. add, The Front of  
a Denticule is in Breadth 3'. their Inter-  
stice 2'.  
P. 246. in the Table of Ecphoræ,  
for 1<sup>m</sup>. 8'  $\frac{1}{4}$  read 1<sup>m</sup>. 22'  $\frac{3}{4}$   
for 1 8  $\frac{1}{2}$  read 1 23  
for 1 9  $\frac{3}{4}$  read 1 24  $\frac{1}{4}$   
P. 256. in the Margin, for Plate XIX. FIG. 3.  
read Plate XVI. FIG. 5.

## DIRECTIONS to the *Book-binder*.

| Plate   | fronting  | Page         |
|---------|-----------|--------------|
| 1       | - - - - - | 26           |
| 2       | - - - - - | 46           |
| 3       | - - - - - | 74           |
| 4       | - - - - - | 122          |
| 5       | - - - - - | 142          |
| 6       | - - - - - | 150          |
| 7       | - - - - - | 156          |
| 8       | - - - - - | 174          |
| 9       | - - - - - | 184          |
| 10      | - - - - - | <i>ibid.</i> |
| 11      | - - - - - | 192          |
| 12      | - - - - - | 196          |
| 13      | - - - - - | 198          |
| 14      | - - - - - | 204          |
| 15      | - - - - - | 208          |
| 16      | - - - - - | 212          |
| 17      | - - - - - | <i>ibid.</i> |
| 18 & 19 | - - - - - | at the End.  |

N. B. The clean Paper on the left Side of the Cuts must  
not be cut off, for the Plate must ly without the Book.





# Trigonometry.

## PART I.

*Of the Construction of Tables of natural Sines, Tangents and Secants.*

### DEFINITIONS.

Plate I.  
FIG. 1.

I. **A** *N Arch* is any Part of the Periphery of a Circle, as  
A B.

II. A *Degree* is the 360<sup>th</sup> Part of the Periphery; hence a *Semicircle* contains 180°, and a *Quadrant* 90°.

III. A *Minute* is the 60<sup>th</sup> Part of a Degree, and a *Second* the 60<sup>th</sup> Part of a Minute; and so on in Thirds and Fourths, &c.

N. B. Arches are the Measure of Angles, so A B is the Measure of the Angle at the Center A O B. i. e. If A B be an Arch of 35°. A O B is said to be an Angle of 35°. Hence, the Sine, Tangent, or Secant of any Arch, may be called the Sine, Tangent, or Secant of the Angle whose Measure that Arch is.

A

IV. The

## 2 Trigonometry. PART I.

IV. The *Chord* or Subtense of any Arch, is a right Line joyning the Extremities of that Arch.

V. The *Complement* of any Arch, is so much as it wants of  $90^\circ$ .

VI. The right *Sine* of any Arch, is Half the Chord of Double that Arch. So  $AD = \frac{1}{2} AC$  is the right Sine of  $AB$ . Or the right Sine is a Line drawn from one Extremity of the Arch, perpendicular to a Diameter that passes through the other Extremity of that Arch, so  $AD \perp HB$ , is the Sine of  $AB$ .

N. B. As every Chord  $AC$  subtends Two Arches  $AHC$ ,  $ABC$ . the one greater, the other less than a Semicircle; so every Sine  $AD$  subtends Two Arches,  $AGH$ ,  $AB$ . the one greater, the other less than a Quadrant; and these Arches  $AGH$ ,  $AB$ . are Complements of each other to a Semicircle.

VII. A *Versed Sine* is so much of the Diameter as is intercepted between the right Sine and the Periphery; so  $DB$  is the versed Sine of the Arch  $AB$ . and  $DH$  is the versed Sine of the Arch  $AGH$ .

VIII. The *Co-Sine* is the Sine of the Complement; so  $AF$  is the Co-Sine of  $AB$ . for it is the Sine of its Complement  $AG$ .

N. B. In the Parallelogram  $DF$  the Side  $DO = AF$  the Co-Sine of  $AB$ . Hence,

The

# PART I. *Trigonometry.*

3

The Co-Sine of any Arch is so much of the Radius as lies betwixt the Center and the right Sine of that Arch.

IX. The *Whole Sine*, or Sine of  $90^\circ$ . is the Radius of the Circle; so  $GO$  is the Sine of the Quadrant  $GB$ .

X. The *Tangent* is a Line touching the Circle at one Extremity of the Arch, and continued till it meet another Line passing through the Center and other Extremity of that Arch; so  $BE$  is the Tangent of  $AB$ .

XI. The *Secant* is that right Line  $OE$ , which limits the Tangent  $BE$ .

XII. *Co-Tangent* and *Co-Secant* are the Tangent and Secant of the Complement, as  $GL$ ,  $OL$ .

XIII. *Tables* of natural Sines, Tangents and Secants, are numeral Tables expressing the natural Sines, Tangents and Secants of the several Arches, from one Minute to  $90^\circ$ . in Parts, of which Parts the Radius contains a certain Number taken at Pleasure, suppose 10000000; so if the Radius  $GO$  be supposed to be divided into 10000000 equal Parts,  $AD$  the Sine of  $AB (=35^\circ)$  shall contain 5735764 of these Parts.



Plate I.  
FIG. 2.

## PROPOSITION I.

*Given DE the Sine of any Arch DC.  
sought AD its Co-Sine.*

In the right angled Triangle DEB;

<sup>a</sup> 47. e. 1.  
<sup>b</sup> Def. 8.

$$BD^a - DE^a = BE^b = AD^a. \text{ Q. E. F.}$$

Cor. Hence,  $BC - BE = EC = {}^v DC$ .

Plate I.  
FIG. 2.

## PROP. II.

*Given the Sine of any Arch, to find  
the Sine of Half or Double that  
Arch.*

1. Given  $DE = {}^s DC$ , sought  $DF =$   
<sup>a</sup> 47. e. 1.  $\frac{1}{2} {}^s DC. \frac{1}{2} \sqrt{DE^a + EC^a} = \frac{1}{2} \sqrt{DC^a} = \frac{1}{2} DC^b =$   
<sup>b</sup> 3. e. 3.  $DF. \text{ Q. E. F.}$

<sup>c</sup> Def. 6.

<sup>d</sup> 4. e. 6.

2. Given  $DF$ , sought  $DE$ . The Tri-  
angles  $DEC$ ,  $BFC$  are similar, for  $\angle^{\text{le}} c$   
is common, and  $DEC$ ,  $BFC^c$  are right;  
therefore  $BC. BF^d :: DC. DE$ , that is,  
 $R. \sigma :: 2 S. DE. \text{ Q. E. F.}$

Plate I.  
FIG. 2.

## PROP. III.

*Given KI, DE the Sines of Two Ar-  
ches KD, DC, sought KM, the Sine  
of their Sum KC.*

Com-



# PART I. Trigonometry. 5

Complete the Parallelogram PINM, the Triangles BDE, BIN, BOM, OIK, OIP, KIP are all similar; therefore  $BD. DE^2 :: BI. IN = PM.$  and  $BD. BE^2 :: ^2 4. e. 6.$   $KI. KP.$  and  $KP + PM = KM.$  Q. E. F.

## PROP. IV.

Plate I.  
FIG. 2.

*Given KM, DE the Sines of Two Arches, sought KI the Sine of their Difference.*

$BE. DE^2 :: BM. OM$  and  $OM$  given,  $OK$  is known; so  $BD. BE^2 :: OK, KI.$

N. B. The Sines of lesser Arches are (at least Physically) as their Arches; for in the Beginning of a Quadrant, the Sine and Arch<sup>b</sup> being both perpendicular to the Diameter, <sup>b</sup> Def. 6. they must co-incide, and consequently their Ratio's are equal.

## PROP. V.

Plate I.  
FIG. 3.

*The Difference of the Sines of Two Arches equally distant from 60° is equal to the Sine of the common Distance.*

Make DE, CF the Sines of Two Arches AD, AC. which Two Arches are equally distant from  $60^\circ = AB.$   $DB = BC$  is the common Distance, whose Sine is  $DI = IC.$  Draw  $CG^a \parallel FE,$  so is <sup>a</sup> 31. e. 1.

DG

# 6 Trigonometry. PART I.

$DG = DE - CF =$  the Difference of the Sines  $DE, CF$ . I say that  $DG = DI$ . Draw  $CM$ .

The  $\Delta^{les} MID, MIC$  have the  $\angle^{les}$  at  $I^b$  right,  $ID = IC$  and  $MI$  common, therefore  $IMC^c = IMD^d = BKN^e = 30^\circ$ . Therefore  $CMD = 60^\circ^f = MDI = MCI$ , therefore  $MD^g = DC$ . But  $DG = \frac{1}{2}MD$ ; for in the  $\Delta^{les} CGD, CGM$  the  $\angle^{les}$  at  $G$  are right, the  $\angle^{le} GDC = GMC$  and  $GC$  is common, therefore  $DG^h = GM = \frac{1}{2}DM = \frac{1}{2}DC = DI$ . *Q. E. F.*

*Cor.* Hence, if the Sines of Arches to  $60^\circ$  are known, the rest to  $90^\circ$  are found by Addition only; for  $^s43^\circ + ^s17^\circ = ^s60^\circ + 17^\circ = ^s77^\circ$ .

Plate I.  
FIG. 3.

## PROP. VI.

*IF Two Arches are equally distant from  $30^\circ$ . the Square of the Sine of their common Distance is  $\frac{1}{3}$  of the Square of the Difference of the Sines of these Two Arches.*

Make the Arches  $ND, NC$  equally distant from  $NB = 30^\circ$  whose Sines are  $LD, PC$ , then is  $GC$  the Difference of the Sines; I say  $DI_q = \frac{1}{3}GC_q$ . For seeing

# PART I. Trigonometry. 7

seeing  $DI = DG$ , in the right angled Triangle  $CGD$ , we have  $DI_q$  (i. e.  $DG_q$ )  $+ GC_q^a = DC_q^b = 4DI_q$ , therefore  $GC_q^c = 3DI_q$  and  $\frac{GC_q}{3} = DI_q$ . *Q. E. F.*  
*Cor.* Hence, if the Sines to  $30^\circ$  are known, the rest to  $60^\circ$  are easily found, for  $13^\circ \times \sqrt{3 \times 17^\circ} = 47^\circ$ .

## PROP. VII.

*I*F any Quantity  $aec$  ( $=60$ ) be divided by Two Quantities,  $ac=10$  and  $ec=15$ , the Divisors and Quotes shall be reciprocal-ly proportional.  $\left\{ \begin{matrix} ac \\ ec \end{matrix} \right\} aec \left( \begin{matrix} e \\ a \end{matrix} \right)$   
 $ac.ec :: a.e$  for  $ac \times e = ec \times a = aec$   
 $10.15 :: 4.6$  for  $10 \times 6 = 15 \times 4 = 60$

## PROP. VIII.

*T*He Radius is equal to the Chord of  $60^\circ$ .

For the Radius is the Side of an Hexagon inscribed in a Circle, and consequently it subtends  $\frac{1}{6}$  of the Periphery or  $360^\circ$ . and  $\frac{360^\circ}{6} = 60^\circ$ . *Q. E. D.*

## PROP.



## PROP. IX.

**TO** construct the Tables of Sines.

Let the Radius be supposed to be divided into any Number of Parts, suppose 10000000 (equal to it is <sup>a</sup> the Subtense of  $60^\circ$ ) the Half of that Subtense is the Sine of <sup>b</sup>  $30^\circ =$  <sup>s</sup> A = 5000000. A-

gain,  $\frac{1}{2}$  that Arch of  $30^\circ$  is  $15^\circ =$  B, whose Sine is found by  $2^d$ . Thus Bisecting the Arches continually, and finding their

Sines (by  $2^d$ ) we shall at the Twelfth Division find that the Sines are as their Arches, *i. e.* the last Sine is just  $\frac{1}{2}$  the preceeding Sine, as the last Arch is  $\frac{1}{2}$  the preceeding Arch.

Now dividing the Arch of  $30^\circ$  into Parts, whereof the Arch P is one,

|     | 0  | ' | "  | ''' | ''' | ''' |
|-----|----|---|----|-----|-----|-----|
| A = | 30 | . |    |     |     |     |
| B = | 15 | . |    |     |     |     |
| C = | 7  | . | 30 | .   |     |     |
| D = | 3  | . | 45 | .   |     |     |
| E = | 1  | . | 52 | .   | 30  | .   |
| F = |    | . | 56 | .   | 15  | .   |
| G = |    | . | 28 | .   | 7   | .   |
| K = |    | . | 14 | .   | 3   | .   |
| L = |    | . | 7  | .   | 1   | .   |
| M = |    | . | 3  | .   | 30  | .   |
| N = |    | . | 1  | .   | 45  | .   |
| P = |    | . |    | .   | 52  | .   |

the

I. PART I. *Trigonometry.* 9

the Quote or Number of such Parts contained in  $30^\circ$  is 2048.

But the same Arch of  $30^\circ$  divided into Minutes gives 1800', wherefore  
 $1800'. 2048^a :: P : 1'^b :: \text{Sine } P. \text{ Sine } 1'$  <sup>a 7<sup>b</sup></sup>  
<sub>b N. B. 4.</sub>

The Sine of one Minute being given, that of  $2'$  is known by  $2^d$ . that of  $3'$  by  $3^d$ ; as also that of 4, 5, 6, &c. to  $30^\circ$ , thence to  $60^\circ$  by  $6^{\text{th}}$ , and thence to  $90^\circ$  by  $5^{\text{th}}$ .

The Table of *Sines* being made, the Tables of *Tangents* and *Secants* are easily constructed by the Two following Propositions.

PROP. X.

Plate I.  
FIG. 4.

*The Co-Sine of any Arch AB is to its Sine as the Radius to the Tangent of that Arch.*

'Tis plain,  $DC. CB^2 :: DA. AF.$  <sup>a 4. e. 6.</sup>

B

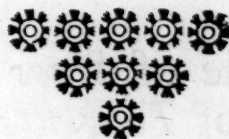
PROP.

Platc I.  
FIG. 4.

PROP. XI.

**T**He Radius is a Mean proportional  
betwixt the Co-Sine and Secant  
of any Arch.

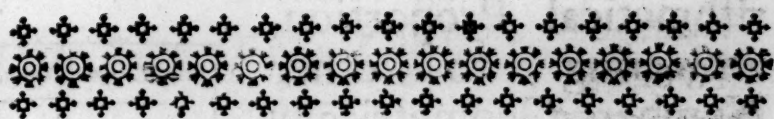
For  $DC.DA::DB.DF.$  Q. E. D.




---

Tri-





# Trigonometry.

---

## PART II.

---

*Of the Construction of Tables of Logarithms.*

I. **F**OUR Numbers are said to be in Arithmetical Proportion, when the Difference of the First and Second is equal to that of the Third and Fourth.

So  $3.5:::16.18.$  and  $7.4:::15.12.$

II. Three Numbers are in continued Arithmetical Proportion, when the Difference of the First and Second is equal to that of the Second and Third.

So  $3.5.7\equiv,$  and  $8.5.2\equiv,$  and  $0.1.2\equiv.$

# 12 Trigonometry. PART II.

III. *Logarithms* are Numbers in Arithmetical Proportion, answering to other Numbers Geometrically proportional: So 0, 1, 2, 3, 4, &c. are the Logarithms of the Geometrical Proportionals, 1, 10, 100, 1000, 10000, &c.

| Proport. Geo. | Arith. |
|---------------|--------|
| 1             | 0      |
| 10            | 1      |
| 100           | 2      |
| 1000          | 3      |
| 10000         | 4      |
| 100000        | 5      |
| 1000000       | 6      |

## PROP. I.

*IF Four Numbers be in Arithmetical Proportion, the Sum of the Extremes is equal to that of the middle Terms.*

If 9 . 11 :: 12 . 14, then is  $9 + 14 = 11 + 12 = 23$ .

## PROP. II.

*IF Three Numbers are in Arithmetical Proportion, the Sum of the Extremes is double the middle Term.*

If 8 . 13 . 18 ::, then is  $8 + 18 = 2 \times 13 = 26$ .

## PROP.

PROP. III.

*The Sum of the Logarithms of Two entire Numbers is equal to the Logarithm of their Product, when the Logarithm of Unity is 0.*

Make the Numbers 100 and 1000, whose Logarithms are 2 and 3, let the Logarithm of their Product (100000) be  $x$ . then  $1. 100^a :: 1000. 100000.$  <sup>*a* Def. Mult.</sup>  
 and  $0. 2^b :: 3. x.$  Therefore,  $0 + x^c$  <sup>*b* Def. 3.  
*c* 1 *b*.</sup>  
 $= 2 + 3 = 5 = x.$  Q. E. D.

*Cor.* Hence, Addition of Logarithms is the same as Multiplication of their correspondent Numbers.

PROP. IV.

*The Difference of the Logarithms of Two entire Numbers is equal to the Logarithm of their Quote, when the Logarithm of Unity is 0.*

Make the Numbers 100 and 100000, whose Logarithms are 2 and 5, and make the Logarithm of their Quote (1000) =  $x$ . Then  $1. 1000^a :: 100.$  <sup>*a* Def. Div.</sup>  
 $100000,$  and  $0. x^b :: 2. 5.$  <sup>*b* Def. 1.</sup>

Therefore  $0 + 5^c = x + 2 = 5,$  and <sup>*c* 1 *b*.</sup>  
 transposing 2,  $x = 5 - 2 = 3.$  Q. E. D.  
 Hence,



14 *Trigonometry.* PART II.

Hence, Subtraction of Logarithms is the same as Division of their correspondent Numbers.

PROP. V.

*THE Logarithm of any Number is  $\frac{1}{2}$  the Logarithm of its Square, and  $\frac{1}{3}$  the Logarithm of its Cube, when the Logarithm of Unity is 0.*

Make the Number 100, whose Logarithm is 2, and make the Logarithm of its Square  $x$ , and of its Cube  $z$ .

It is plain 1. 100. 1000  $\div$ , therefore  
<sup>a 2 b.</sup> 0. 2.  $x$   $\div$ , consequently  $0 + x^a = 2 \times$   
 $2. = x$ .

Likewise 1. 100  $::$  10000. 1000000,  
<sup>b 1 b.</sup> therefore 0. 2  $::$  4.  $z$ . and  $0 + z^b =$   
 $2 + 4 = 2 \times 3 = z$ .

*Schol.* After the same Manner we may demonstrate, that the Logarithm of the biquadrate Root is  $\frac{1}{4}$  the Logarithm of the Biquadrate, and universally the Logarithm of the Root of any Power is equal to the Logarithm of that Power divided by its Exponent.

*Cor.* The Logarithm of any Power is found by multiplying the Logarithm  
of

**PART II. Trigonometry.** 15  
of the Root by the Exponent of the Power.

**PROP. VI.**

*TO find a Mean Proportional Geometrical betwixt Two Numbers given.*

The Square Root of their Product is the Mean, by 20 of *Euclid's* Seventh Book.

**PROP. VII.**

*TO find a Mean Proportional Arithmetical betwixt Two Numbers given.*

Half their Sum <sup>a</sup> is the Mean sought. <sup>a</sup> Prop. 2.

*N. B.* Because there are Eight intermediate Numbers betwixt 0 and 10, and Eighty nine betwixt 10 and 100, &c.

and yet no intermediate Number betwixt 0 and 1, or betwixt 1 and 2, therefore we shall joyn Seven Cyphers to the Logarithms first proposed, so the Logarithm of 1 will be 0.0000000, the Logarithm of 10 will be 1.0000000, that of 100 will be 2.0000000, &c. and then the Logarithms first placed will be the Characteristicks of the several Logarithms,

i. e. the Marks by which we may know what Class of Numbers the Logarithm belongs to; so 0 is the Characteristick of Units, 1 of Tens, 2 of Hundreds, &c.

| Propor. Geo. | Propor. Arith. |
|--------------|----------------|
| 1            | 0.0000000      |
| 10           | 1.0000000      |
| 100          | 2.0000000      |
| 1000         | 3.0000000      |
| 10000        | 4.0000000      |
| 100000       | 5.0000000      |
| 1000000      | 6.0000000      |

**PROP.**

| Prop. Geo. |            | Logar.    |
|------------|------------|-----------|
| A          | 1.0000000  | 0.0000000 |
| C          | 3.1622777  | 0.5000000 |
| B          | 10.0000000 | 1.0000000 |
| <hr/>      |            |           |
| B          | 10.0000000 | 1.0000000 |
| D          | 5.6234132  | 0.7500000 |
| C          | 3.1622777  | 0.5000000 |
| <hr/>      |            |           |
| B          | 10.0000000 | 1.0000000 |
| E          | 7.4989421  | 0.8750000 |
| D          | 5.6234132  | 0.7500000 |
| <hr/>      |            |           |
| B          | 10.0000000 | 1.0000000 |
| F          | 8.6596432  | 0.9375000 |
| E          | 7.4989421  | 0.8750000 |
| <hr/>      |            |           |
| B          | 10.0000000 | 1.0000000 |
| G          | 9.3057204  | 0.9687500 |
| F          | 8.6596432  | 0.9375000 |
| <hr/>      |            |           |
| G          | 9.3057204  | 0.9687500 |
| H          | 8.9768713  | 0.9531250 |
| F          | 8.6596432  | 0.9375000 |
| <hr/>      |            |           |
| G          | 9.3057204  | 0.9687500 |
| I          | 9.1398170  | 0.9609375 |
| H          | 8.9768713  | 0.9531250 |
| <hr/>      |            |           |
| I          | 9.1398170  | 0.9609375 |
| K          | 9.0579777  | 0.9570312 |
| H          | 8.9768713  | 0.9531250 |
| <hr/>      |            |           |
| K          | 9.0579777  | 0.9570312 |
| L          | 9.0173333  | 0.9550781 |
| H          | 8.9768713  | 0.9531250 |
| <hr/>      |            |           |
| L          | 9.0173333  | 0.9550781 |
| M          | 8.9970796  | 0.9541015 |
| H          | 8.9768713  | 0.9531250 |

## PROP. VIII.

*TO find the Logarithm of any Number.*

Make the Number 9, because it is between 1 (A) and 10 (B) find c a Mean proportional Geometrical betwixt A and B ( augmenting each by as many Cyphers as their Logarithms contain ) and, because c is less than 9.0000000, find d a Mean proportional betwixt B and c, and betwixt B and



and D, another Mean E; and so on, till you come to G, which is greater than 9.0000000. Then betwixt G and F (which F is next to 9.0000000, of all these that are less than it) find another Mean H, and another I betwixt G and H; betwixt which I (because it is greater than 9.0000000) and H find a Mean K, and so on till at the 26<sup>th</sup> Time you find 9.0000000 (i. e. 9) whose Logarithm is easily found.

C For

| <i>Prop. Geo.</i> |           | <i>Logar.</i> |
|-------------------|-----------|---------------|
| L                 | 9.0173333 | 0.9550781     |
| N                 | 9.0072008 | 0.9545898     |
| M                 | 8.9970796 | 0.9541015     |
| <hr/>             |           |               |
| N                 | 9.0072008 | 0.9545898     |
| O                 | 9.0021388 | 0.9543457     |
| M                 | 8.9970796 | 0.9541015     |
| <hr/>             |           |               |
| O                 | 9.0021388 | 0.9543457     |
| P                 | 8.9996088 | 0.9542236     |
| M                 | 8.9970796 | 0.9541015     |
| <hr/>             |           |               |
| O                 | 9.0021388 | 0.9543457     |
| Q                 | 9.0008737 | 0.9542846     |
| P                 | 8.9996088 | 0.9542236     |
| <hr/>             |           |               |
| Q                 | 9.0008737 | 0.9542846     |
| R                 | 9.0002412 | 0.9542541     |
| P                 | 8.9996088 | 0.9542236     |
| <hr/>             |           |               |
| R                 | 9.0002412 | 0.9542541     |
| S                 | 8.9999250 | 0.9542388     |
| P                 | 8.9996088 | 0.9542236     |
| <hr/>             |           |               |
| R                 | 9.0002412 | 0.9542541     |
| T                 | 9.0000831 | 0.9542465     |
| S                 | 8.9999250 | 0.9542388     |
| <hr/>             |           |               |
| T                 | 9.0000831 | 0.9542465     |
| V                 | 9.0000041 | 0.9542427     |
| S                 | 8.9999250 | 0.9542388     |
| <hr/>             |           |               |
| V                 | 9.0000041 | 0.9542427     |
| X                 | 8.9999650 | 0.9542428     |
| S                 | 8.9999250 | 0.9542188     |
| <hr/>             |           |               |
| V                 | 9.0000041 | 0.9542427     |
| Y                 | 8.9999845 | 0.9542421     |
| X                 | 8.9999650 | 0.9542428     |

| <i>Prop. Geo.</i> |           | <i>Logar.</i> |
|-------------------|-----------|---------------|
| V                 | 9.0000041 | 0.9542427     |
| Z                 | 8.9999943 | 0.9542422     |
| Y                 | 8.9999845 | 0.9542421     |
| <hr/>             |           |               |
| V                 | 9.0000041 | 0.9542427     |
| &                 | 8.9999992 | 0.9542424     |
| Z                 | 8.9999943 | 0.9542422     |
| <hr/>             |           |               |
| V                 | 9.0000041 | 0.9542427     |
| A                 | 9.0000016 | 0.9542425     |
| &                 | 8.9999992 | 0.9542424     |
| <hr/>             |           |               |
| A                 | 9.0000016 | 0.9542425     |
| B                 | 9.0000004 | 0.9542425     |
| &                 | 8.9999992 | 0.9542424     |
| <hr/>             |           |               |
| B                 | 9.0000004 | 0.9542425     |
| C                 | 8.9999998 | 0.9542425     |
| &                 | 8.9999992 | 0.9542424     |
| <hr/>             |           |               |
| B                 | 9.0000004 | 0.9542425     |
| D                 | 9.0000000 | 0.9542425     |
| C                 | 8.9999998 | 0.9542425     |

For as we found  
c a Geometrick  
Mean betwixt A  
and B, so an Ari-  
thmetick Mean  
betwixt their  
Logarithms will  
be the Loga-  
rithm of c, and  
so at last the  
Logarithm of  
9.0000000 will  
be found to be  
0.9542425.

In the same  
Manner are  
found the Loga-  
rithms of Num-  
bers betwixt 1

and 9, betwixt 10 and 100, &c. But  
the Logarithms of compound Num-  
bers are more easily found by Addi-  
tion, by PROP. III.

### PROP. IX.

**T**O find the Logarithm of an entire  
Number that is greater than 10000.  
Make

## PART II. *Trigonometry.* 19

Make the Number 3567894, cut off from the right Hand so many Figures, as what remains on the left may be found in the Tables, as here 894.

To the Logarithm of 3567 add that of 1000, so you have 6.552303 for the Logarithm of 3567000, which being yet less than the Logarithm of 3567894, find 122 the Difference betwixt the Logarithm of 3568 and that of 3567; then say, If 1000 give 122, what will 894 give? The Answer, 109 added to the Logarithm of 3567000 gives 6.552412, for the Logarithm of 3567894.

| N.            | Logar.         |
|---------------|----------------|
| 3567          | 3.552303       |
| 1000          | 3.000000       |
| <hr/> 3567000 | <hr/> 6.552303 |
| 894           | + 109          |
| <hr/> 3567894 | <hr/> 6.552412 |
| 3568          | 3.552425       |
| 3567          | 3.552303       |



$$1000 : 122 :: 894$$

$$122$$

$$\begin{array}{r} 122 \\ \hline 1788 \\ 1788 \\ 894 \end{array}$$

$$1000) 109068 (109.$$

*Cor.* Hence may be constructed the Tables of *Logarithmick* or *Artificial* Sines, and by it the Tables of artificial Tangents and Secants.

From what has been already said the Two following Problems are easily understood.

### PROB. I.

**T**O find the Logarithm of any Number.

1. Find the Number in the Column under *N.* and, in the immediately next Column opposite to the Number, you have the Logarithm.

2. If the Number extends to Thousands, and your Tables have 0, 1, 2, 3, &c. over Head, find the first Three Figures in the Left-hand Column, opposite

## PART II. *Trigonometry.* 21

posite to it, and directly below the Fourth Figure (*i. e.* the Units of your Number) you have its Logarithm.

3. If your Number is compounded of an Integer and a decimal Fraction, find its Logarithm as if it was Integers, but allow it the Characteristick of the Integer only.

4. If the Logarithm of a decimal Fraction is demanded, find its Logarithm as if it was Integers, and place it *negative* with a Characteristick, as many Units below 10, as the first significant Figure of your Decimal has Places from Unity. Thus the Logarithm of

$$7964 = 3.9011313$$

$$796.4 = 2.9011313$$

$$79.64 = 1.9011313$$

$$7.964 = 0.9011313$$

$$.7964 = -9.9011313, \text{ or } \bar{9}.9011313$$

$$.07964 = -8.9011313, \text{ or } \bar{8}.9011313$$

$$.007964 = -7.9011313$$

Ec.

Ec.

PROB.

## PROB. II.

*TO find the Number answering to any Logarithm.*

In Tables that have 0, 1, 2, 3, &c. over Top, seek the Logarithm, or that next less than it, in the Tables, omitting the Index; the Number in the Left-hand Column, with the Figure directly above the Logarithm, is the Number sought, which must be ordered according to the Index. Thus the Number answering to

$$\begin{aligned}
 3.7982362 &= 6284 \\
 2.7982362 &= 628.4 \\
 1.7982362 &= 62.84 \\
 0.7982362 &= 6.284 \\
 -9.7982362 &= .6284 \\
 -8.7982362 &= .06284
 \end{aligned}$$

If the Logarithm is not exactly found in the Tables, and Five Places are required, find the first Four Places as before; with the common Difference in the Right-hand Column enter the Table of proportional Parts, and above



# PART II. *Trigonometry.* 23

bove the Difference of the Logarithm given, and its next less Logarithm in the Tables, you have the Figure in the Fifth Place.

*Ex.* In 2.543612, the Logarithm next less is 543571, its Number is 3496; the common Difference is 124, and  $543612 - 543571 = 41$ , above which, in the Table of proportional Parts, is 3, so the absolute Number is 34963, and the Index being 2, it is 349.63.

*N. B.* To find the Logarithm of a vulgar Fraction, subtract the Logarithm of the Denominator from that of the Numerator. So the Logarithm of  $\frac{3}{4} = 0.477121 - 0.602060 = \bar{9}.875061 = \text{Logarithm } .75.$

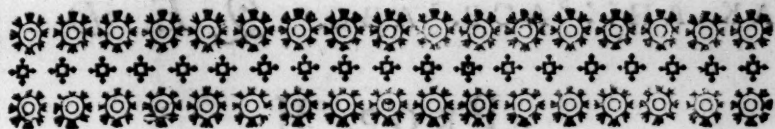


Tri-

have the difference of the logarithm in given, and its next less Logarithm in the Table, you have the figure in the Table.

And in a right-angled Triangle, the Logarithm next less to the given Logarithm is the Logarithm of the Sine, and the difference between it and the given Logarithm is the Logarithm of the Tangent, which is in the Table of proportional Parts, is to the whole Number is added, and the Index being 2, this added.

N. B. To find the Logarithm of a Fraction, take the Logarithm of the Numerator, and subtract from it the Logarithm of the Denominator, the Remainder is the Logarithm of the Fraction.



# Trigonometry.

## PART III.

*Of the Solution of plain Triangles.*

PROP. I.

Plate I.  
FIG. 5, 6, 7.

**I**N any plain Triangle  $ABC$ , the Sides are proportional to the Sines of the opposite Angles.

About the  $\Delta^{le} ABC$  describe a <sup>a</sup> Circle, from the Center  $D$  to the several Sides <sup>b</sup> draw Perpendiculars, which <sup>b</sup> shall bisect the Sides <sup>c</sup> at  $K, I, H$ , and the Arches <sup>d</sup> at  $G, E, F$ , from the Center to the several Angles draw right Lines  $DA, DB, DC$ .  $AK^c = \frac{1}{2} AB$  is the <sup>e</sup> Sine of  $\angle^{le} ADG = \frac{1}{2} \angle^{le} ADB^c$  Def. 6.  $=^f \angle^{le} ACB$ .  $AH^c = \frac{1}{2} AC$  is the Sine of  $\angle^{le} ADE^d = \frac{1}{2} \angle^{le} ADC^f = \angle^{le} ABC$ . And  $D$  it



# 26 Trigonometry. PART III.

<sup>s</sup> 15. c. 5. it is plain, that  $AB.AC^g::\frac{1}{2}AB.\frac{1}{2}AC::$   
 $AK.AH::^sACB.^sABC.$  Q. E. D.

Plate II.  
 FIG. 8, 9, 10.

## PROP. II.

*I*N any right angled  $\Delta^{le} ABC$  right angled at B, making any Side the Radius, the other Two Sides will be Sines, Tangents and Secants.

<sup>a</sup> Def. 10. 1. Make CB the Radius, then is AB the <sup>a</sup> Tangent, and AC the <sup>b</sup> Secant of the Angle c.  
<sup>b</sup> Def. 11.

2. If AB be the Radius, then is CB the <sup>a</sup> Tangent, and AC the Secant of the Angle A.

<sup>c</sup> Def. 6. 3. If CA be the Radius, then is AB the <sup>c</sup> right Sine, and CB the <sup>d</sup> Co-Sine of the Angle c, consequently CB <sup>c</sup> is the right Sine of the Angle A.  
<sup>d</sup> N. B. 3.

Plate II.  
 FIG. 11.

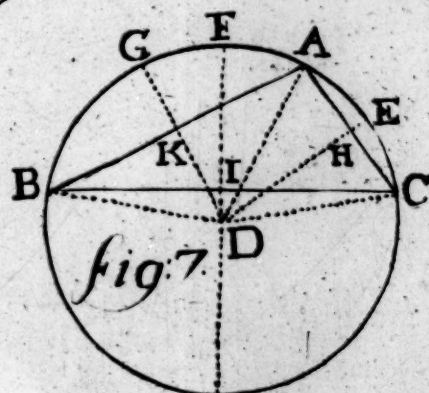
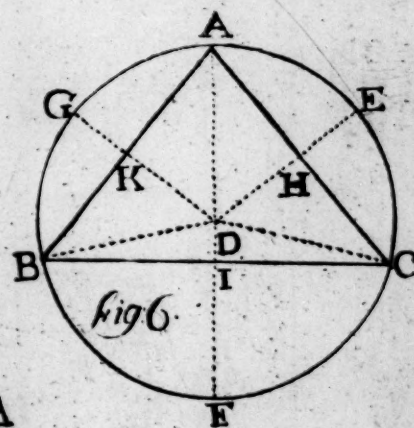
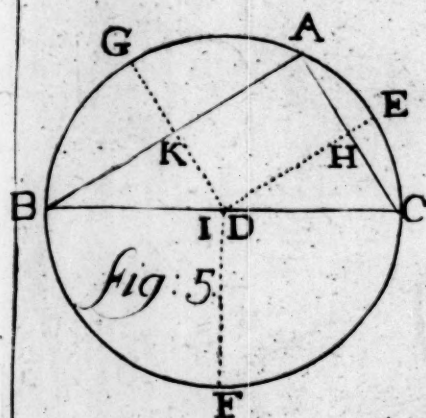
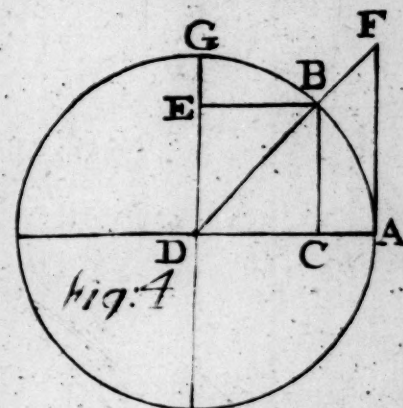
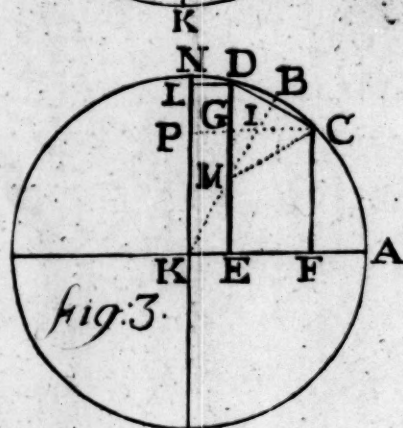
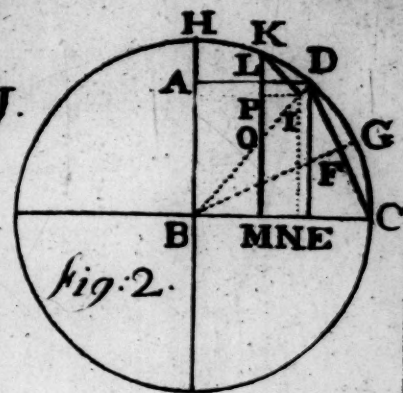
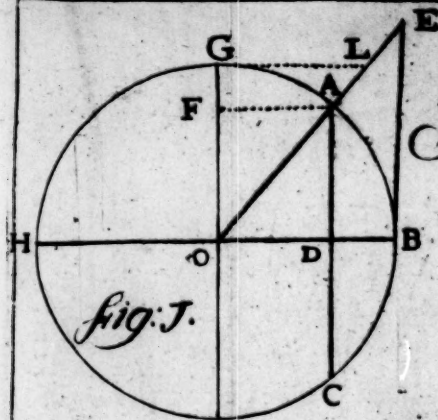
## PROP. III.

*I*N any Triangle ACE, as the Sum of the Sides is to the Difference of the Sides, so is the Tangent of  $\frac{1}{2}$  the Sum of the opposite Angles to the Tangent of  $\frac{1}{2}$  their Difference,  $CB+BA.$

$$CB-BA::T\frac{1}{2}A+ACB.T\frac{1}{2}A-ACB.$$

Pro-

Plate J.



Produce AB to H, making BH = BC;  
cut off BI = AB, so is AH the Sum,  
and IH the Difference of the Sides.

Joyn CH, draw BE<sup>a</sup> ⊥ CH, that BE<sup>a</sup> <sup>12. e. 1.</sup>  
shall bisect<sup>b</sup> CH, and<sup>c</sup> the Angle CBH, <sup>b Cor. 3. e. 3.</sup>  
so is CBE =  $\frac{1}{2}$  CBH<sup>d</sup> =  $\frac{1}{2}$  the Sum of <sup>c 8. e. 1.</sup>  
the Angles A + ACB, draw<sup>e</sup> BD and<sup>e</sup> <sup>32. e. 1.</sup>  
IG each || AC, then is DBH<sup>f</sup> = A the <sup>29. e. 1.</sup>  
greater Angle, and CBD<sup>f</sup> = ACB the  
lesser Angle: Take HF = CD, joyn BF:  
in the Δ<sup>les</sup> CBD, HBF; CB<sup>g</sup> = BH, <sup>g Construct.</sup>  
CD<sup>g</sup> = HF, and the ∠<sup>le</sup> BCD<sup>h</sup> = BHF; <sup>h 5. e. 1.</sup>  
therefore HBF<sup>i</sup> = CBD<sup>f</sup> = ACB, conse- <sup>i 4. e. 1.</sup>  
quently DBF = DBH - HBF<sup>f</sup> = A -  
ACB the Difference of the Angles:  
And, because BD<sup>i</sup> = BF, BE bisects<sup>b</sup>  
BDF, and the Angle<sup>c</sup> DBF; therefore  
the ∠<sup>le</sup> DBE =  $\frac{1}{2}$  the Difference of the  
Angles =  $\frac{1}{2}$  A - ACB.

Because IG<sup>g</sup> || DB || AC, and AB<sup>g</sup> =  
BI; therefore CD<sup>k</sup> = DG<sup>g</sup> = HF, and, <sup>k 2. e. 6.</sup>  
taking away FG, which is common,  
DF =<sup>l</sup> GH, and consequently  $\frac{1}{2}$  GH = <sup>l Ax. 3.</sup>  
 $\frac{1}{2}$  DF = DE.

On B, as a Center with the Radius  
BE, describe a Circle, then is EC the  
<sup>m</sup> Tangent of the ∠<sup>le</sup> CBE<sup>d</sup> =  $\frac{1}{2}$  the <sup>m Def. 10.</sup>  
Sum, and ED = the <sup>m</sup> Tangent of the

D 2

∠<sup>le</sup> DBE



28 *Trigonometry.* PART III.

$\angle^{\text{le}} \text{DBE} = \frac{1}{2}$  the Difference of the Angles. And  $\text{HA} \cdot \text{HI}^{\text{n}} :: \text{HC} \cdot \text{HG}^{\text{o}} :: \frac{1}{2} \text{HC} \cdot \frac{1}{2} \text{HG} :: \text{EC} \cdot \text{ED}$ . *Q. E. D.*

Platc II.  
FIG. 12.

PROP. IV.

*I*F from D, the Vertex of any Triangle ADE, a Line DB be drawn perpendicular to AE the Base; I say, the Base AE, is to the Sum of the Sides  $\text{AD} + \text{DE}$ , as the Difference of the Sides  $\text{AD} - \text{DE}$  to the Difference of the Segments of the Base  $\text{AB} - \text{BE}$ .

Upon D, as a Center with the Radius DE the lesser Side, describe a Circle cutting the Base AE at I, and the Side AD at P, produce AD till it meet the Periphery at c.

<sup>a</sup> Def. 15. Because  $\text{DC}^{\text{a}} = \text{DE} = \text{DP}$ , and  $\text{BI}^{\text{b}} = \text{BE}$ , 'tis plain AC shall be the Sum of the Sides, AP their Difference, and AI the Difference of the Segments of the Base. But  $\text{AE} \times \text{AI}^{\text{c}} = \text{AC} \times \text{AP}$ ,  
<sup>c</sup> 37. e. 3. wherefore  $\text{AE} \cdot \text{AC} :: \text{AP} \cdot \text{AI}$ . *Q. E. D.*  
<sup>d</sup> 16. e. 6.

FIG. 13, 14,  
15.

In the following Proportions for the Solution of plain Triangles; for right angled Triangles take the Triangle  
ABC,

# PART III. Trigonometry. 29

ABC, whose right Angle is B, and for oblique angled Triangles, take the Two Triangles DEF, IGK, and let IGK be divided into Two right angled Triangles by the Perpendicular IL.

## Solution of right angled Triangles.

|     | Given.                          | Sought.       | Proportions.     |
|-----|---------------------------------|---------------|------------------|
| a 1 | 1. AC.AB                        | ACB           | $AC.R^a::AB.^sC$ |
| b 2 | 2. $\angle^le A.\angle^le C.AB$ | BC            | $R.AB^b::^sA.BC$ |
| c 3 | 3. BA.BC                        | $\angle^le A$ | $BA.R^b::BC.^sA$ |
| d 4 | 4. $\angle^le A.\angle^le C.AC$ | CB            | $R.AC^a::^sA.BC$ |

## Solution of oblique angled Triangles.

|   | Given.                       | Sought.          | Proportions.                                                             |
|---|------------------------------|------------------|--------------------------------------------------------------------------|
| 5 | $\angle^le D.\angle^le E.DF$ | DE               | $^sE.DF^a::^sF.DE$                                                       |
| 6 | DE.EF. $\angle^le F$         | $\angle^le D$    | $DE.^sF^a::EF.^sD$                                                       |
| 7 | DE.EF. $\angle^le E$         | $\angle^les D.F$ | $DE \div EF.DE - EF^c::T \frac{1}{2} D \div F$<br>$.T \frac{1}{2} D - F$ |
| 8 | GI.IK.GK                     | $\angle^le GIK$  | $GK.IK \div IG^d::IK - IG$<br>$.KL - LG$<br>$IG.R::GL.^sGIL, \&c.$       |

N. B. 1. From all the Angles given  $^c$  the Ratio's of the  $^c$  1 b. Sides may be found, but not the Sides themselves.

2. In Case Sixth it must be known if the Angle sought D be greater or less than a right Angle, for every Sine has  $^c$  N. B. 6. two Angles (the one greater the other less than a Quadrant) and only one Angle being given, that sought may be either greater or less than a right Angle, and yet have the same Sine.

3. One

# 30 Trigonometry. PART III.

3. One Angle given, the Sum of the other Two is its Complement to  $180^\circ$ .

4.  $\frac{1}{2}$  the Sum  $\pm \frac{1}{2}$  the Difference of any Two Quantities is  $=$  greater and  $\frac{1}{2}$  the Sum  $- \frac{1}{2}$  the Difference of any Two Quantities is  $=$  lesser.

Make  $a$  the greater,  $b$  the lesser Quantity.

$$\text{Add. } \begin{cases} \frac{1}{2} \text{ Sum} = \frac{1}{2} a + \frac{1}{2} b. \\ \frac{1}{2} \text{ Diff.} = \frac{1}{2} a - \frac{1}{2} b. \end{cases}$$

$$\text{Substr. } \begin{cases} \frac{1}{2} \text{ Sum} = \frac{1}{2} a + \frac{1}{2} b. \\ \frac{1}{2} \text{ Diff.} = \frac{1}{2} a - \frac{1}{2} b. \end{cases}$$

$$\frac{1}{2} \text{ Sum} + \frac{1}{2} \text{ Diff.} = a = \text{the greater.}$$

$$\frac{1}{2} \text{ Sum} - \frac{1}{2} \text{ Diff.} = b = \text{the lesser.}$$



Tri-





# Trigonometry.

## PART IV.

*Of the Solution of spherique Triangles.*

### DEFINITIONS.

I. **A** Globe or Sphere is a solid Body, conceived to be generated by the Rotation of a Semicircle  $AEB$  about its Diameter  $AB$ .

II. The Center of the Sphere is the same with that of the generating Semicircle  $D$ .

*Cor.* Hence, all the right Lines drawn from the Center to the Surface of the Sphere are equal, as  $DE = DC = DL = DF$ .

III. The

# 32 Trigonometry. PART IV.

III. The Axis of the Sphere is the Diameter of the generating Semi-circle  $AB$ .

IV. The Poles of the Sphere are the Extremities of its Axis  $A, B$ .

Plate X.  
FIG. 1.

## LEMMA I.

*IF Two Triangles  $ABC, DEF$ , right angled at  $B$  and  $E$ , have Two Sides  $AC, AB$  in one, equal to Two Sides  $DF, DE$  in the other, the remaining Sides  $BC, EF$  shall be equal.*

<sup>a</sup> 47. c. I.  
<sup>b</sup> Hyp.

For  $BC_q^a = AC_q - AB_q^b = DF_q - DE_q^a = EF_q$ , therefore  $BC = EF$ .  $\square$   
 $E. D.$

Plate X.  
FIG. 2.

## LEMMA II.

*IF a Sphere is cut by any Plan, the Section is a Circle.*

1. Let the Plan  $EKQ$  pass through the Center  $c$ , all the Lines drawn from  $c$  to the Periphery <sup>a</sup> are equal. <sup>a</sup> Cor. Def. 2.  
*Ergo, &c.*

2. If the Plan  $FOM$  does not pass through  $c$ , draw  $CD$  perpendicular to the Plan  $FOM$ , so  $CDF, CDO, CDM$  are

# PART IV. *Trigonometry.* 33

are <sup>d</sup> right Angles  $CF$ ,  $CO$ ,  $CM$  <sup>e</sup> are <sup>d</sup> <sub>3</sub> Def. e.  
equal, therefore <sup>e</sup>  $DF = DO = DM$  ; <sup>II.</sup> <sub>1.</sub> Lem. 1.  
therefore  $FOM$  is a Circle, whose  
Center is  $D$ . Q. E. D.

*Cor. 1.* Hence the Diameter of a  
Circle passing through the Center of  
the Sphere, is equal to the Diameter  
of the generating Circle, and is it self  
a Diameter of the Sphere.

2. The Diameter of a Circle, that  
does not pass through the Center of  
the Sphere, is the Chord of an Arch  
of the generating Circle, or of a Cir-  
cle equal to it ; and therefore

3. Circles, whose Plans pass through  
the Center of the Sphere, are the  
greatest of all Circles on the Surface  
of the Sphere, and are therefore cal-  
led *Great Circles*, and all other Cir-  
cles are called *Lesser Circles*.

4. All great Circles have the Cen-  
ter of the Sphere for their common  
Center, therefore they bisect the  
Sphere, and are all equal amongst  
themselves.

5. A right Line drawn from the  
Center of the Sphere, perpendicular

E

to



# 34 Trigonometry. PART IV.

to the Plan of a lesser Circle, passes through its Center.

V. The Axis of a Circle is a right Line drawn through its Center, perpendicular to its Plan, and limited by the Surface of the Sphere on both Sides.

VI. The Poles of a Circle are the Extremities of its Axis.

*Cor.* Hence, every lesser Circle has the same Poles and Axis with its parallel great one.

VII. A spherique Triangle is contained under the Arches of Three great Circles, as  $\triangle AEC$ .

VIII. A spherique Angle is the Inclination of the Peripheries of Two great Circles, and is the same with the Inclination of the Plans of these Circles, as  $\angle^{\text{le}} EAC = \angle^{\text{le}} EDC$ .

Plate II.  
FIG. 16.

## PROP. I.

*Great Circles*  $AEB$ ,  $ACB$  *bisect each other.*

<sup>a</sup> 3. e. 11.

<sup>b</sup> *Cor. Def. 5.*

For their common Section <sup>a</sup> is a right Line  $AB$  passing through <sup>b</sup> the Center of both Circles, and consequently

# PART IV. Trigonometry. 35

quently 'tis a Diameter to both. Q.  
E. D.

## PROP. II.

Plate II.  
FIG. 16.

*A Great Circle AFBE is  $90^\circ$  distant from its Pole c.*

Because  $AD^a = DC$ , and  $DC^b \perp AD$ , <sup>a Cor. Def. 2.</sup>  
therefore  $AC = 90^\circ$ , and consequently <sup>b Def. 6.</sup>  
 $CB^c = 90^\circ$ . The same Way  $CE$ ,  $CF^e$  <sup>1 b.</sup>  
are Quadrants. Q. E. D.

Cor. The Distance of any lesser Circle from its Poles  $= 90^\circ \pm$  its Distance from its parallel great Circle.

## PROP. III.

Plate II.  
FIG. 16.

*If a great Circle ECF be described from the Pole A, the Arch EC, lying betwixt AC and EA, which Form the Angle at A, is the Measure of that Angle EAC.*

For  $AC^a = AE = 90^\circ$ , therefore the <sup>a 2. b.</sup>  
Angle  $ADC^b = 90^\circ^b = ADE$ , where- <sup>b N. B. 3.</sup>  
fore  $CDE^b = EC^c =$  to the Inclination <sup>P. I.</sup>  
of the Two Plans ACD,  $AED^d = \angle^le$  <sup>c Def. 6. e.</sup>  
<sup>d Def. 8.</sup> EAC. Q. E. D.

Plate II.  
FIG. 16.

## PROP. IV.

*A Great Circle ACB cutting another AEBF, makes the Angles deinceps equal.*

<sup>a</sup> 3 b.

<sup>b</sup> 1 b.

<sup>c</sup> N. B. 3.  
P. I.

For  $EAC$ ,  $CAF^a =$  to the Arches  $EC$ ,  $CF^b =$  to a Semicircle or  $180^\circ$   
 $^c =$  to Two right Angles.

Plate II.  
FIG. 16.

## PROP. V.

*The Angles at the Vertex  $EAC, FAL$ , are equal.*

<sup>a</sup> 3 b.

<sup>b</sup> N. B. 3.  
P. I.

<sup>c</sup> 15. E. II.

For  $\angle^{lc} EAC^a = EC^b = EDC^c =$   
 $FDL^b = LF^a = FAL$ . Q. E. D.

## PROP. VI.

*Angles  $EAC, EBC$ , at a Semicircle's Distance, are equal.*

<sup>a</sup> 6 b.

<sup>b</sup> 3 b.

For  $EC^a = EDC$  the Inclination of the Two Plans<sup>b</sup>, is the Measure of both Angles.

PROP.



PROP. VII.

*T*WO Triangles, having one Angle and the Sides forming that Angle equal, are every Way equal.

PROP. VIII.

*T*WO Triangles, having one Side and the adjacent Angles equal, are every Way equal.

PROP. IX.

*E*quilateral Triangles are likewise equiangular.

PROP. X.

*I*soceles Triangles have the Angles both above and below the Base, equal.

PROP. XI.

*I*F the Angles at the Base are equal, the Triangle is an Isoceles.

In

# 38 Trigonometry. PART IV.

In this and the Four preceeding, the Demonstration is the same as in plain Triangles.

## PROP. XII.

*ANy Two Sides of a Triangle taken together are greater than the Third.*

For as in a plain Surface, a right Line is the nearest Distance betwixt Two Points, so in a spherique Surface, the Arch of a great Circle.

Plate II.  
FIG. 16.

## PROP. XIII.

*ANy Side of a Triangle AEC is less than a Semicircle.*

Produce AE, AC till they meet at B, then are AEB, ACB Semicircles, and 'tis plain  $AEB \sqsupset AE$ , and  $ACB \sqsupset AC$ ; also  $ECF \sqsupset EC$ . Q. E. D.

Plate II.  
FIG. 16.

## PROP. XIV.

*ALL the Sides of a Triangle are less than a Circle.*

For

# PART IV. *Trigonometry.* 39

For  $EC^a \supset EB + BC$ , therefore  $AE^a \supset 12 b.$   
 $+ AC + EC \supset AEB + ACB^b. i. e. \supset^b 1 b.$   
 a Circle. Q. E. D.

## PROP. XV.

Plate II.  
FIG. 16.

*IN any Triangle AEC, the greater Side subtends the greater Angle AEC.*

Because  $AEC^a \supset EAC$ , make  $AEd^a$  Hyp.  
 $= EAC$ , then is  $Ed^b = Ad$  and  $AC =^b 11 b.$   
 $Ad + dc = Ed + dc^c \supset EC. Q.^c 12 b.$   
 E. D.

*Schol.* The Converse is demonstrated as in 19. e. I.

## PROP. XVI.

Plate II.  
FIG. 16.

*IN any Triangle AEC, if the Sum of the Sides  $AE + EC$  be  $\supset, =$  or  $\supset$  than a Semicircle, the internal Angle  $EAC$ , at the Base  $AC$ , will be  $\supset, =$  or  $\supset$  than the external and opposite Angle  $ECB$ , and therefore the Sum of the Angles  $A + ACE \supset, =$  or  $\supset$  than Two Rights.*

If  $AE + EC \supset, =$  or  $\supset AEB$ , then is  $^a 6 b.$   
 $EC \supset, =$  or  $\supset EB$ , and  $EBC (^a = A)^b \supset^c 15 b.$   
 $=$  or  $^c 10 b.$



# 40 Trigonometry. PART IV.

= or  $\supset$  ECB, and consequently  $A + ACE \supset$ , = or  $\supset$  ECB + ACE, which are<sup>d</sup> equal to Two Rights.

<sup>a</sup> 4 <sup>b</sup>.

Plate II.  
FIG. 16.

## PROP. XVII.

**I**N the spherique Triangle ABC, if from A, B and C, as Poles, Arches of great Circles be described forming a Triangle DEF, I say,

1. The Sides of that Triangle DEF are the Complements of the Angles of the Triangle ABC to Semicircles.

2. The Angles of the Triangle DEF are the Complements of the Sides of the Triangle ABC to Semicircles.

Upon A, B, C, as Poles, describe the great Circles FIEL, TED, VFSD, then is  $AE^a = 90^{\circ a} = BE$ , therefore E<sup>a</sup> is the Pole of AB; also  $BD^a = 90^{\circ a} = CD$ , therefore D<sup>a</sup> is the Pole of BC. Again,  $AF^a = 90^{\circ} = CF$ , therefore F is<sup>a</sup> the Pole of AC.

<sup>a</sup> 2 <sup>b</sup>.

<sup>b</sup> Ax. 2.

<sup>a</sup> 1 <sup>b</sup>.

<sup>d</sup> 3 <sup>b</sup>.

I.  $FG^a = 90^{\circ a} = EL$ , then  $FE^b = GL^c = \text{Complement IG, i. e. } ^d \angle^{\text{le}} A \text{ to } 180^{\circ}$ .

$TE = 90^{\circ} = DN$ , then  $DE^b = TN^c = \text{Complement RN, i. e. } ^d \angle^{\text{le}} B \text{ to } 180^{\circ}$ .

FV

# PART IV. Trigonometry. 41

$FV = 90^\circ = DO$ , then  $DF^b = VO^c =$   
Complement  $os$ , *i. e.*  $^d \angle^l c$  to  $180^\circ$ .

2.  $\angle^l E^d = IR = AI + BR - AB =$   
Two Quadrants  $- AB$ .

$\angle^l D^d = ON = BN + CO - BC =$   
Two Quadrants  $- BC$ .

$\angle^l F^d = SG = GA + CS - AC =$   
Two Quadrants  $- AC$ . *Q. E. D.*

## PROP. XVIII.

*Equiangular Triangles are likewise  
equilateral.*

For their Complements <sup>a</sup> are equi- <sup>a 17 b.</sup>  
lateral, and therefore <sup>b</sup> equiangular, <sup>b 9 b.</sup>  
wherefore the Triangles themselves  
are <sup>a</sup> equilateral. *Q. E. D.*

## PROP. XIX.

Plate II.  
FIG. 17.

*The Three Angles of any spherique  
Triangle are  $\angle$  than Two and  $\angle$   
than Six Rights.*

For 6 Rights  $- A - B - C^a = DF^a$  <sup>a 17 b.</sup>  
 $+ FE + DE^b \supset 4 R$ ; and transpo- <sup>b 14 b.</sup>  
sing  $- A - B - C$  we have  $6 R \supset$   
 $4 R + A + B + C$ , and taking  $4 R$  from

F

both

42 *Trigonometry.* PART IV.

both Sides, we have  $2 R \supset A + B + c$ . *Q. E. D.*

The Second Part is plain; for the external and internal Angles together are equal to Six right Angles. *Ergo, &c.*

Plate II.  
FIG. 16.

PROP. XX.

*A* Circle *FCEL* passing through *L*, *c*, the Poles of another great Circle *AFBE* makes with it right Angles.

<sup>a</sup> Hyp.

<sup>b</sup> Def. 6.

<sup>c</sup> 18. e. 11.

For it passes through <sup>a</sup> *Lc*, which is <sup>b</sup> right to the Plan *AFBE*, therefore *FCEL* <sup>c</sup> is perpendicular to that Plan.

*Q. E. D.*

*Cor. 1.* A great Circle passing through the Poles of a lesser Circle is perpendicular to that lesser Circle, for it passes through its Axis which is perpendicular to its Plan.

2. A great Circle passing through the Poles of a lesser Circle bisects that lesser Circle, as passing through its Axis, and consequently through its Center.

PROP.



PROP. XXI.

Plate II.  
FIG. 16.

**I**F a great Circle  $FCEL$  be right to another  $AFBE$ , it shall pass through its Poles  $C, L$ .

Because its Plan passes <sup>a</sup> through  $D$ , <sup>a</sup> Def. 5. and is right <sup>b</sup> to the Plan  $AFBE$ ;  $CD$ , <sup>b</sup> Hyp. which is <sup>c</sup> right to the same Plan, will <sup>c</sup> Def. 6. be <sup>d</sup> in its Plan. *Ergo, &c.* <sup>d</sup> 4. e. II.

PROP. XXII.

Plate II.  
FIG. 18.

**I**F to a great Circle  $AFBE$ , from a Point  $R$ , which is not its Pole, there be drawn Arches of great Circles  $RA, RB, RT, RV$ , that Arch  $RA$ , which passes through  $C$  the Pole of  $AFBE$ , is the greatest; its Complement to a Semicircle  $RB$  is the least; and any Arch  $RT$ , which is nearer to  $RA$ , is greater than that  $RV$  which is farther removed from it, and the Arches shall make obtuse Angles towards  $RA$ .

$BRCA^a LAFB$ . Draw  $RS$  <sup>a</sup> to the <sup>20</sup>  $h$ . Plan  $AFB$ , that  $RS$  <sup>b</sup> shall fall upon  $AB$  <sup>c</sup> <sup>38</sup>  $e. II.$  the common Section and common <sup>c</sup> <sup>1</sup>  $b$ . Diameter of the Circles  $ACB, AFB$ ,  
F 2 joyn

# 44 Trigonometry. PART IV.

joyn the Right Lines RA, RB, RT, RV, and ST, SV.

In the  $\Delta^{les}$  RSA, RST, RSV, RSB,  
<sup>d</sup> right angled at s;  $RA_q^c = RS_q + SA_q^f \sqsubset RS_q + ST_q^c$ , *i. e.*  $RT_q$ , there-  
<sup>a</sup> 3. e. 11.  
<sup>e</sup> 47. e. 1.  
<sup>f</sup> 7. e. 3.  
<sup>g</sup> 28. e. 3.  
fore RA  $\sqsubset$  RT, and the Arch RA  $\sqsubset$  RT; the same Way  $RT_q^c = RS_q + ST_q^f \sqsubset RS_q + SV_q^c$ , *i. e.*  $RV_q^f \sqsubset RS_q + SB_q^c$ , *i. e.*  $RB_q$ . *Ergo*, &c.

2. Because c is the Pole of the  
<sup>a</sup> 20 b.  
great Circle AFBE, therefore  $CTA^h$  shall be a right Angle, and consequently RTA shall be obtuse. Q. E. D.

N. B. Two Arches or Angles are of the same Affection, when they are at once  $\sqsubset$  or  $\supset$  than  $90^\circ$ .

Plate II.  
FIG. 18.

## PROP. XXIII.

*IN any spherique Triangle right angled at A, the Sides containing the right Angle, are of the same Affection with their opposite Angles.*

<sup>a</sup> 21 & 2 b.  
<sup>b</sup> 20 b.  
<sup>c</sup> 22 b.  
For if  $Ac = 90^\circ$  then <sup>a</sup> c is the Pole of the Circle AFBE, and  $CTA^b$  a right Angle  $= 90^\circ^b = CVA$ . If  $AR \sqsubset 90^\circ$ , then <sup>c</sup> are RTA, RVA obtuse Angles. But if  $AX \supset 90^\circ$ , then are XTA, XVA,

less

# PART IV. Trigonometry. 45

less than  $\angle CTA$ ,  $\angle CVA^b$ , *i. e.* less than right Angles. *Q. E. D.*

## PROP. XXIV.

Plate II.  
FIG. 18.

*IF the Two Sides or Angles of a right angled Triangle be of the same Affection, the Hypothenufe will be less than  $90^\circ$ .*

In the Triangle  $BRV$ , because  $BR^a$  *Hyp.*  $\angle 90^\circ$ , *i. e.*  $BC$  and  $BV^a \angle 90^\circ$ , *i. e.*  $BF$ , then is  $RV^b \angle RF$ , which  $RF^c = b_{22}^b$  *a Quadrant.*  $c_{22}^b$

2. Because  $AR \angle AC$ , *i. e.*  $90^\circ$ , and  $AV \angle AF$ , *i. e.*  $90^\circ$ , therefore  $RV^b \angle RF$ , *i. e.*  $90^\circ$ . *Q. E. D.*

## PROP. XXV.

Plate II.  
FIG. 18.

*IF the Two Sides of a right angled Triangle be of a different Affection, the Hypothenufe will be greater than a Quadrant.*

If  $AR \angle AC$ , *i. e.*  $90^\circ$ , and  $AT \angle AE$ , *i. e.*  $90^\circ$ , then is  $RT^a \angle RF^b$ , *i. e.*  $90^\circ$ .  $a_{22}^b$  *Q. E. D.*  $b_{22}^b$

PROP.



Plate II.  
FIG. 18.

# PROP. XXVI.

<sup>a</sup> 23 h.

**I**F the Hypothenuſe be  $\angle$  or  $\rhd$  than  $90^\circ$ , the Sides, and <sup>a</sup> conſequently their oppoſite Angles, will accordingly be of the ſame or different Affections.

It follows from the Two preceeding Propoſitions.

Plate II.  
FIG. 19.

# PROP. XXVII.

**I**N any Triangle ABC, if the Angles  $B, C$ , at the Baſe, be of the ſame Affection, the Perpendicular AD ſhall fall within the Triangle.

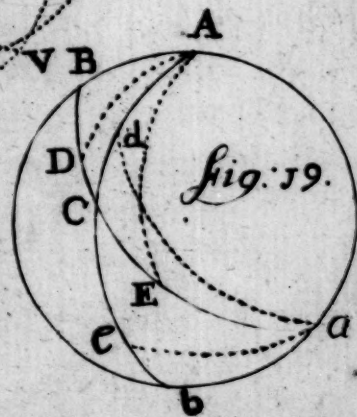
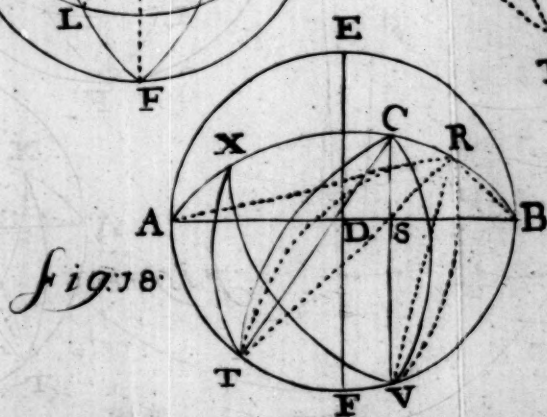
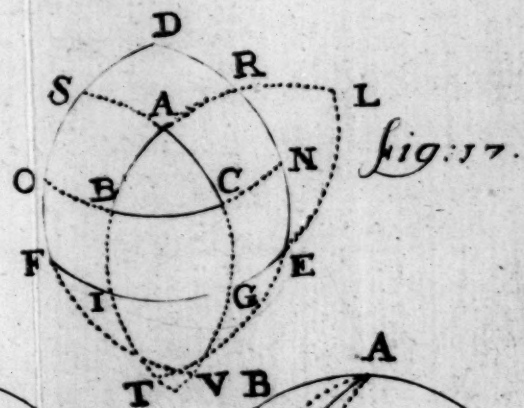
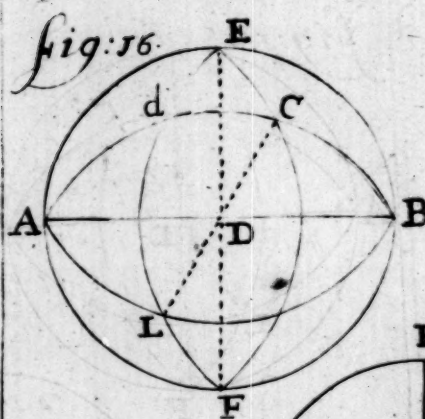
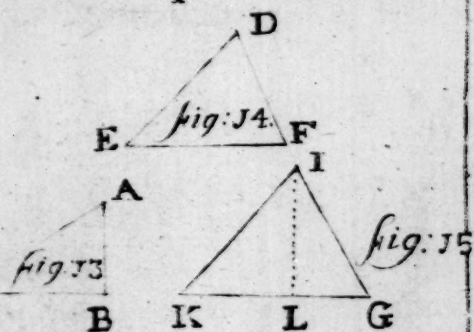
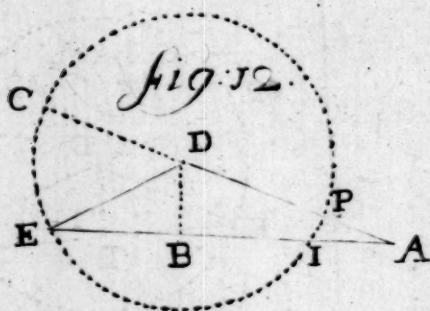
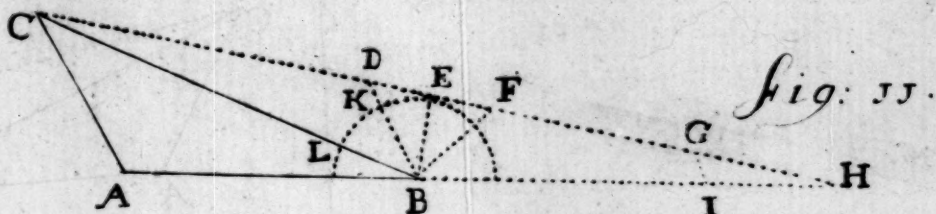
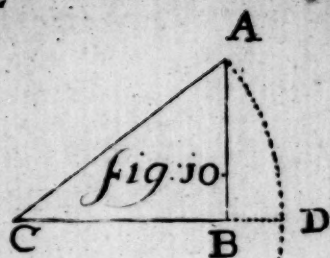
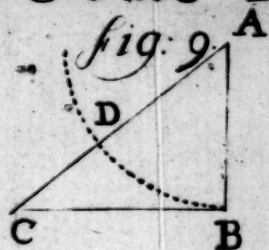
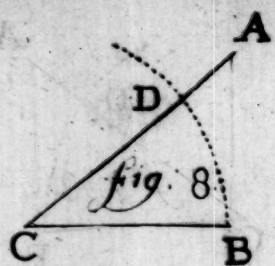
2. But if the Angles  $b, c$ , be of different Affections, the Perpendicular Ad ſhall fall without the Triangle.

<sup>a</sup> 23 h.

1. In the Firſt Caſe, if AD be not the Perpendicular, let AE, without the  $\Delta^{le}$ , be it; then in the right angled  $\Delta^{les}$  AEC, AEB,  $AE^a$  is affected as  $\angle^{le}$  ACE, and the ſame  $AE^a$  is affected as ABE, wherefore ABE is affected as ACE *contra Hyp.*

2. In

# Plate 2



PART IV. *Trigonometry.* 47

2. In the  $\Delta^{le} abc$ , if  $Ad$  be not the Perpendicular, let  $ae$  within the  $\Delta^{le}$  be it. Then in the right angled Triangles  $aeb$ ,  $aec$ ,  $ae^2$  is affected as  $b$ , and  $ae^2$  is affected as  $ACE$ ; wherefore  $b$  and  $ACE$  are of the same Affection *contra Hyp.* Q. E. D.

PROP. XXVIII.

Plate III.  
FIG. 20.

**I**N the Triangles  $BAC$ ,  $BHE$  right angled at  $A$  and  $H$ , and having the same acute Angle  $B$ , the Sines of the Hypothenuses  $BE$ ,  $BC$  are as the Sines of the Perpendiculars  $HE$ ,  $CA$ .

$BT$  is the common Section of the Plans of the Circles  $BCT$ ,  $BHT$ , then  $ER$ ,  $CP$  the Sines of  $BE$ ,  $BC^a$  are paral-<sup>a Def. 6.</sup>  
lel;  $EF$ ,  $CD$  the Sines of  $HE$ ,  $AC$  be-<sup>P. I. & 23.</sup>  
ing<sup>b</sup> perpendicular to the Plan  $BHT$ ,<sup>b 4. c. II.</sup>  
are likewise<sup>c</sup> parallel, therefore the<sup>c 6. c. II.</sup>  
Plans  $REF$ ,  $PCD^d$  are parallel, and<sup>d 15. c. II.</sup>  
 $RF^e \parallel PD$ , and consequently the Tri-<sup>e 16. c. II.</sup>  
angles  $REF$ ,  $PCD^f$  are similar, and  $RE$ .<sup>f 10. c. II.</sup>  
 $CP^g :: EF.CD.$  Q. E. D. <sup>g 4. c. 6.</sup>

PROP.



Plate III.  
FIG. 20.

## PROP. XXIX.

*I*N the same Triangles, the Sines of the Bases  $BH$ ,  $BA$  are as the Tangents of the Perpendiculars  $HE$ ,  $AC$ .

$HK$ ,  $AQ$  the Sines of  $BH$ ,  $BA$  are <sup>a</sup> parallel, and  $HG$ ,  $AI$  the Tangents of  $HE$ ,  $AC$  being <sup>b</sup> perpendicular to the Plan  $BHT$ , are likewise <sup>c</sup> parallel, therefore the Plan  $KHG$  is <sup>d</sup>  $\parallel$   $QAI$  and  $KG$  <sup>e</sup>  $\parallel$   $QI$ , therefore the  $\Delta^{les}$   $KHG$ ,  $QAI$  <sup>f</sup> are similar, and  $KH.QA^g :: HG.AL$ .  
*Q. E. D.*

Plate III.  
FIG. 21.

## PROP. XXX.

*I*N the Triangle  $BAC$ , right angled at  $A$ , if the Sides  $BA$ ,  $BC$ ,  $AC$  be continued, and on the Poles  $B$ ,  $C$  great Circles be described, as  $EFG$ ,  $IHG$ ;  $EF$  shall be the Measure of the Angle  $B$  and  $IH$  the Measure of  $HCI = ACB$ . The Angles at  $A$  and  $E$  being right,  $I$  shall be the Pole of  $BAE$ ;  $CD$  the Complement of  $AC$ , and  $DF$  the Complement of  $EF$ , i. e. of the Angle  $ABC$ .

The

# PART IV. *Trigonometry.* 49

The Angles at F and I being right, G is the Pole of BCI, so FD is the Complement of DG, therefore  $DG = EF = \angle^c ABC$ . Also CF is the Complement of FI, and of BC, therefore  $FI = BC$ . CD is the Complement of DH, then  $DH = AC$ . 'Tis plain too that AE is the Complement of AB, and GH that of HI, or ACB.

Hence follow the Sixteen Cases of right angled spherique Triangles.

In the Table here adjoyned the First Column gives the Order of the Cases, the Second has the *Data*, and the Fourth the *Quæsitæ* in general Terms, the Third and Fifth give the same *Data* and *Quæsitæ* in the Triangle BAC, the Sixth has the Proportions for the Resolution of the several Cases, the Seventh the Affections of the Sides and Angles, and the last the Triangles which afford the Demonstration of the Proportions.

G

PROP.

Plate III.  
FIG. 22.

## PROP. XXXI.

**I**N oblique angled spherique Triangles  $BCD$ ,  $bcd$ , if a Perpendicular  $CA$ , from the vertical Angle  $c$  to the Base  $BD$  or  $bd$ , divide the Triangles into Two other Triangles  $BCA$ ,  $DCA$ , or  $bca$ ,  $dca$ , right angled at  $A$  or  $a$ , the Co-Sines of the  $\angle^{les}$  at the Base  $B$  and  $D$ , ( $b$  and  $d$ ) are as the Sines of the vertical  $\angle^{les}$   $BCA$ ,  $DCA$ ,  $bca$ ,  $dca$ .

<sup>a</sup> 30 <sup>b</sup>.  
Case 6.  
invert.

For  $\sigma_B . {}^sBCA^a :: \sigma_{AC} . R^a :: \sigma_D . {}^sDCA$ . Q. E. D.

Plate III.  
FIG. 22.

## PROP. XXXII.

**T**He Co-Sines of the Sides  $BC$ ,  $DC$ ,  $bc$ ,  $dc$ , are as the Co-Sines of the Bases  $BA$ ,  $DA$ ,  $ba$ ,  $da$ .

<sup>a</sup> 30 <sup>b</sup>.  
Case 4.

For  $\sigma_{BC} . \sigma_{BA}^a :: \sigma_{AC} . R^a :: \sigma_{DC} . \sigma_{DA}$ . Q. E. D.

PROP.



PROP. XXXIII.

Plate III.  
FIG. 22.

*The Sines of the Bases*  $BA, DA, ba, da$ , *are reciprocally as the Tangents of*  $\angle^{les} B, D, b, d$ , *at the Base*  $BD, bd$ .

Because  ${}^sBA.R^a :: {}^tAC.{}^tB$ , therefore  ${}^sBA \times {}^tB^b = {}^tAC \times R$ . And because  ${}^sDA.R^a :: {}^tAC.{}^tD$ , therefore  ${}^sDA \times {}^tD^b = {}^tAC \times R$ , therefore  ${}^sBA \times {}^tB^b = {}^sDA \times {}^tD^b$ .  $Q. E. D.$

PROP. XXXIV.

Plate III.  
FIG. 22.

*The Tangents of the Sides*  $BC, DC, bc, dc$ , *are reciprocally as the Co-Sines of the vertical*  $\angle^{les} BCA, DCA, bca, dca$ .

Because  ${}^tBC.{}^tAC^a :: R.\sigma BCA$ , therefore  ${}^tBC \times \sigma BCA^b = {}^tAC \times R$ , and because  ${}^tDC.{}^tAC^a :: R.\sigma DCA$ , therefore  ${}^tDC \times \sigma DCA^b = {}^tAC \times R$ , therefore  ${}^tBC \times \sigma BCA^b = {}^tDC \times \sigma DCA^b$ , and  ${}^tBC.{}^tDC^b :: \sigma DCA.\sigma BCA$ .  $Q. E. D.$

Plate III.  
FIG. 23.

PROP. XXXV.

*The Sines of the Sides BC, DC, bc  
dc are as those of their opposite  
Angles D, B.*

<sup>a</sup> 30 h.

Case 2.

<sup>b</sup> 16. c. 6.

<sup>c</sup> Ax. 1.

<sup>d</sup> 14. c. 6.

Because  $^sBC \cdot R^a :: ^sAC \cdot ^sB$ , therefore  
 $^sBC \times ^sB^b = R \times ^sAC$ , and because  $^sDC$ .  
 $R^a :: ^sAC \cdot ^sD$ , therefore  $^sDC \times ^sD^b = R$   
 $\times ^sAC$ , therefore  $^sBC \times ^sB^c = ^sDC \times ^sD$ ,  
and  $^sBC \cdot ^sD^d :: ^sDC \cdot ^sB$ . Q. E. D.

Plate X.  
FIG. 3.

LEMMA I.

*IF a Cone ABDC be cut through the  
Axis Ae, the Section shall be a  
Triangle.*

<sup>a</sup> 3. c. 11.

For BC, the common Section of  
the secant Plan with that of the Base<sup>a</sup>,  
is a right Line, and AB, AC are right  
Lines, by Def. 18. e. 11.

Plate X.  
FIG. 3.

LEMMA II.

*IF a Cone ABDC be cut by a Plan  
gmo parallel to the Base, the Se-  
ction is a Circle.*

Make

# PART IV. Trigonometry. 53

Make  $ABC$  the Triangle through the Axis;  $go$ , the common Section of its Plan with the secant Plan, is <sup>a</sup> 4. e. 6. parallel to  $BC$ , as are also  $mn$ ,  $ef$  the common Sections of any other Triangle  $AFe$ , with the Base and secant Plan. So  $Be.gn^a::Ae.An^a::ec.no$ , but  $Be=ec$ , therefore  $gn^b=no$ ; <sup>b</sup> 14. e. 5. the same Way  $Be.gn^a::Ae.An^a::fe.mn$ , but  $Be=fe$ , therefore  $gn=mn=no$ . Ergo, &c. Q. E. D.

Def. A scalenous Cone is said to be cut by a Plan in a subcontrary Position to the Base, when the Plan of the Triangle through the Axis, being perpendicular both to the Plan of the Base and secant Plan, the common Section of the Triangle and secant Plan, cuts off a Triangle equiangular to that through the Axis, but so as the equal Angles ly at opposite Sides of the Axis.

## LEMMA III.

Plate X.  
FIG. 4.

[If a scalenous Cone  $ABDC$  be cut by a Plan  $KMS$ , in a subcontrary Position to the Base, the Section is a Circle.

Draw



# 54 Trigonometry. PART IV.

<sup>a</sup> Lem. 2. Draw the Plan  $gmo \parallel BDC$ , it is a Circle, and  $mn$  the common Section of the Plans  $kms$ ,  $gmo$ <sup>b</sup> is perpendicular to  $go$  and  $ks$ , therefore  $gn \times no = mn_q$ : But in the Triangles  $gnk$ ,  $sno$ , the Angle  $gkn$ <sup>c</sup> =  $ACB$ <sup>d</sup> =  $son$ , and  $gnk$ <sup>e</sup> =  $sno$ , therefore the Triangles  $gnk$ ,  $sno$  are similar, therefore  $gn.nk$ <sup>f</sup> ::  $sn.no$ , and  $nk \times sn$ <sup>f</sup> =  $gn \times no = mn_q$ , therefore  $kms$ <sup>g</sup> is a Circle. Q. E. D.

N. B. If the Eye is situated any where above the Plan of a Circle, Rays from the Eye, to every Point in the Periphery of the Circle, form a Cone whose Base is the Circle, its Vertex the Eye, and its Axis a Line joyning the Eye and Center of the Circle.

2. If the Axis of the Cone is perpendicular to the Plan of its Base, it is a right Cone, otherwise a scalenous Cone.

Plate X.  
FIG. 5.

## LEMMA IV.

**I**F a Sphere touch a Plan  $ag$  at a Point  $D$ , and the perpendicular Diameter  $DO$  be drawn, the Eye at  $o$  shall project any Circle  $OADG$  (passing through the Eye) on the Plan  $ag$  into a right Line.

For seeing the Eye is in the Plan of the Circle to be projected, and the Rays from the Eye to the Plan are right

PART IV. *Trigonometry.* 55

right Lines, the Eye at  $o$  shall see the Point  $A$  on the Plan below at  $a$ ,  $B$  at  $b$ ,  $n$  at  $m$ , &c. in the common Section of the Circle to be projected, and the Plan  $ag$ . *Ergo*, &c. *Q. E. D.*

*Schol.* The Projection of the Circle  $oag$ , is measured from  $D$  on the Line of Semitangents.

For on the Center  $o$ , with the Radius  $oD$ , describe a Circle, 'tis plain  $Dm$  is the Tangent of the Angle  $Dom$ , that is, the Semitangent of  $Dcn$ , or of the Arch  $Dn$ .

LEMMA V.

Plate X.  
FIG. 6, 7.

*Any other Circle not passing through the Eye (whether it be greater or lesser) is projected into a Circle.*

Let  $AB$  be the Diameter of the Circle to be projected on the Plan  $ag$ , Rays from  $o$  to that Circle form a Cone, whose Triangle through the Axis is  $oAB$ ; the Eye at  $o$  sees  $A$  on the Plan,  $ag$  at  $d$ , and  $B$  at  $b$ , so  $db$  is the projected Diameter. Suppose the Cone continued below  $ag$  to the Diameter of the Base  $MN \parallel AB$ , draw  
 $AYQ$

# 56 Trigonometry. PART IV

- <sup>a</sup> 30. c. 3.  $AYQ \parallel ag$ , the Arch  $OA^a = OQ$ , and  
<sup>b</sup> 26. c. 3. the Angle  $OAQ^b = OBA$ , therefore  
<sup>c</sup> 29. c. 1.  $Od b^c = OAQ = OBA^c = ONM$ .

So the Cone, whose Diameter of its Base is  $MN$ , is cut in a subcontrary Position by the Plan through  $db$ , therefore that Section is a Circle. *Q. E. D.*

Plate X.  
FIG. 8.

## PROP. XXXVI.

*I*N any oblique angled spherique Triangle  $ABD$ , if from the Vertex  $A$  to the Base  $BD$  a Perpendicular  $AE$  be drawn, the Semitangent of the Base is to the Semitangent of the Sum of the Sides, as the Semitangent of the Difference of the Sides to the Semitangent of the Difference of the Segments of the Base.

Let the Sphere  $OPDG$  touch a Plan at  $D$ , the Diameter  $DO$  is perpendicular to that Plan.

On the Pole  $A$ , with the Distance  $AB$  the lesser Side, describe the lesser Circle  $FbRm$  cutting the Base  $BD$  at  $b$ , and the Side  $AD$  produced at  $FR$ ; so  $AB = Ab = AR = AF$ , therefore

DF



# PART IV. Trigonometry. 57

DF is the Sum, DR the Difference of the Sides, and Db the Difference of the Segments of the Base, because  $BE^* = bE$ . I say,  $T_{\frac{1}{2}}DB . T_{\frac{1}{2}}DF :: T_{\frac{1}{2}}DR . T_{\frac{1}{2}}Db$ . \* Theor. following.

The Arches DA, DB continued <sup>a</sup>, <sup>a</sup> I b. shall meet at o, and the Eye at o projects DF, DR, DB, Db into the <sup>b</sup> Lem. 4. right Lines Df, Dr, Dy, Dβ, which <sup>a</sup> are the Semitangents of these Arches, but the Points f, r, β, y <sup>c</sup> are <sup>c</sup> Lem. 5. in the Periphery of a Circle, whence  $DY \times D\beta^d = Df \times Dr$ , therefore  $DY .^d 36. e. 3. Df :: Dr . D\beta$ , i. e.  $T_{\frac{1}{2}}DB . T_{\frac{1}{2}}DF :: T_{\frac{1}{2}}Dr . T_{\frac{1}{2}}Db$ . Q. E. D.

Schol. Making the Triangle AbD, where the Perpendicular AE falls without the Triangle, DE is the greater, bE the less Segment of the Base, Db is the Difference, and DB the Sum of these Segments; and 'tis plain that  $T_{\frac{1}{2}}Db . T_{\frac{1}{2}}DF :: T_{\frac{1}{2}}DR . T_{\frac{1}{2}}DB$ .

Theor. In the spherique Isosceles Triangle ABb, the Perpendicular AE bisects the Base Bb.

For AE is perpendicular to <sup>c</sup> BEb. Hyp. and <sup>c</sup> to B ib, therefore their common <sup>c</sup> Cr. 20. Section <sup>c</sup> Bb is perpendicular to the <sup>c</sup> 19. e. 11.  
H Plan

58 *Trigonometry.* PART IV.

Plan of  $AE$ , therefore  $Bb$  is perpendicular to the common Section of the Plans  $BEb$  and  $AE$ , and is consequently <sup>h</sup> bisected by that common Section. *Ergo*, &c.

From the last Six Propositions follow the Resolutions of the Twelve Cases of oblique angled spherique Triangles, which we shall shew in particular in the Two Triangles  $ADC$ ,  $AOC$ , in which  $AB$ ,  $CE$  are Perpendiculars.

RESOLUTION of *spherique Triangles* by Lord Napier's universal Rule.

The ingenious Lord *Napier*, neglecting the right Angle, calls the Five other Parts of a spherique right angled Triangle *Circular Parts*, Two of which are taken simply, *viz.*  $AB$ ,  $BC$ , the other Three are Complements.

The Two given circular Parts, with the Part sought, are either *conjunct*, *i. e.* adjacent to one another, or *disjunct*, where the middle Part is separated

# PART IV. *Trigonometry.* 59

rated from the Extremes by interja-  
cent Parts. So  $A, A C, c$ , as likewise  
 $A, A B, B C$ , are Parts conjunct, and  
 $A B, A C, B C$  are Parts disjunct.

Then, in order to the easy Resol-  
ving all right angled *spherique* Tri-  
angles, he gives the following

RULE. The Sines of the middle  
Part and Radius are reciprocally, as  
the Tangents of the Extremes con-  
junct, and, as the Co-Sines of the Ex-  
tremes, disjunct, likewise reciprocally.

N. B. If the middle Term is sought, begin your Propor-  
tion with the Radius, otherwise begin with the given Ex-  
treme.

This Rules I shall prove from the  
Sixteen Cases of right angled *spherique*  
Triangles, as they are demon-  
strated in our *spherique Trigonome-*  
*try*, PROP. XXX. by Help of the  
following

## LEMMA VI.

See FIG. I.  
*Plan Trig.*

*The Radius is a Mean proportional  
betwixt the Tangent and Co-Tan-*  
*gent of any Arch.*

H 2

For



60 *Trigonometry.* PART IV.

For in the similar  $\Delta^{les}$  OBE, OGL,  
BE, OB :: OG, GL. Q. E. D.

SIXTEEN CASES.

I.  $R. {}^sAC :: {}^sA. {}^sBC.$

II.  ${}^sAC.R :: {}^sBC. {}^sA.$

III.  ${}^sA.R :: {}^sBC. {}^sAC.$

IV.  $\sigma AB.R :: \sigma AC. \sigma BC,$

V.  $R. \sigma BC :: \sigma AB. \sigma AC.$

VI.  $R. {}^sC :: \sigma BC. \sigma A.$

VII.  $\sigma BC.R :: \sigma A. {}^sC.$

VIII.  ${}^sC.R :: \sigma A. \sigma BC.$

N. B. These first Eight Cases are the same as in *sph.*  
*rique Trigonometry*, PROP. XXX.

The other Eight we shall demonstrate from those in that  
PROP. XXX.

IX.  ${}^tA.R :: {}^sAB. {}^tBC.$

For  $R. {}^sAB^a :: {}^tA. {}^tBC.$

$R. {}^tA^b :: {}^sAB. {}^tBC^c :: {}^tA.R.$

unde  ${}^tA.R :: {}^sAB. {}^tBC.$

X.  $R. {}^tA :: {}^tBC. {}^sAB,$

For  ${}^tA. {}^tBC^a :: R. {}^sAB,$

${}^tA.$

<sup>a</sup> 30 h.  
Case 9.

<sup>b</sup> 16. e. 4.  
& Lem. 6.

<sup>a</sup> 30 h.  
Case 10.

# PART IV. Trigonometry. 61

$$^T A . R^b :: ^T B C . ^s A B^c :: R . ^t A .$$

unde  $R . ^t A :: ^T B C . ^s A B .$

XI.  $^T B C . R :: ^s A B . ^t A .$  <sup>a</sup> 30 b.  
Case 11.

For  $^s A B . R^a :: ^T B C . ^T A .$

$$^s A B . ^T B C^b :: R . ^T A^c :: ^t A . R .$$

unde  $^T B C . ^s A B^d :: R . ^t A .$  and  $^T B C^d$  11. c. 5.  
& Cor. 4.  
c. 5.

$$. R^b :: ^s A B . ^t A .$$

XII.  $^t A C . R :: \sigma C . ^T B C .$  <sup>a</sup> 30 b.  
Case 12.

For  $R . \sigma C^a :: ^T A C . ^T B C .$

$$R . ^T A C^b :: \sigma C . ^T B C :: ^t A C . R .$$

unde  $^t A C . R :: \sigma C . ^T B C .$

XIII.  $^T B C . R :: \sigma C . ^t A C .$  <sup>a</sup> 30 b.  
Case 13.

For  $\sigma C . R^a :: ^T B C . ^T A C .$

$$\sigma C . ^T B C^b :: R . ^T A C^c :: ^t A C . R .$$

unde  $^T B C . \sigma C^d :: R . ^t A C .$  and  $^T B C$

$$. R^b :: \sigma C . ^t A C .$$

XIV.  $R . ^t A C :: ^T B C . \sigma C .$  <sup>a</sup> 30 b.  
Case 14.

For  $^T A C . ^T B C^a :: R . \sigma C .$

$$^T A C . R^b :: ^T B C . \sigma C^c :: R . ^t A C .$$

unde  $R . ^t A C :: ^T B C . \sigma C .$

XV.  $^t C . R :: \sigma A C . ^t A .$  <sup>a</sup> 30 b.  
Case 15.

For  $R . \sigma A C^a :: ^T C . ^t A .$

$$R . ^T C^b :: \sigma A C . ^t A^c :: ^t C . R .$$

unde  $^t C . R :: \sigma A C . ^t A .$

XVI. R.

# 62 Trigonometry. PART IV. P

\* 30 b.  
Case 16.

$$\text{XVI. } R. {}^tC :: {}^tA. \sigma AC.$$

$$\text{For } {}^tC. {}^tA^a :: R. \sigma AC.$$

$${}^tC. R^b :: {}^tA. \sigma AC^c :: R. {}^tC.$$

$$\text{unde } R. {}^tC :: {}^tA. \sigma AC.$$

By the same Rule we may have an easy Solution of oblique angled spheric Triangles, resolving the Triangle into Two right angled ones, by a Perpendicular drawn from one Angle to the opposite Side ; for then the Solution is performed by Two Proportions.

N. B. 1. The Perpendicular must always be drawn from the Extremity of a known Side, and opposite to a known Angle.

2. If the Perpendicular falls upon a known Side, and the Fourth Term of the Proportion be greater than the known Side, the Perpendicular falls without the Triangle, otherwise within.

3. If it is drawn from a known Angle, and the Fourth Proportional be greater than the known Angle, the Perpendicular falls without the Triangle, otherwise within.

4. If the Angles at the Base be of the same Affection, the Perpendicular falls within the Triangle, if of different Affections, without.

$$\text{I. } {}^sAC. {}^sD :: {}^sDC. {}^sA.$$

$$\text{II. } {}^sD. {}^sAC :: {}^sC. {}^sAD.$$

$$\text{III. } {}^tD. R :: \sigma DC. {}^tDCB.$$

$${}^tAC. {}^tDC :: \sigma DCB. \sigma ACB.$$

$$DCB \pm ACB = ACD.$$

$$\text{VI. } {}^tD.$$



# V. PART IV. *Trigonometry.* 63

$$\begin{aligned} \text{IV. } {}^t\text{D.R} &:: \sigma\text{DC} . {}^t\text{DCB}. \\ \text{DCA} \curvearrowright \text{DCB} &= \text{ACB}. \\ \sigma\text{ACB} . \sigma\text{DCB} &:: {}^t\text{DC} . {}^t\text{AC}. \end{aligned}$$

$$\begin{aligned} \text{V. } {}^t\text{DC.R} &:: \sigma\text{D} . {}^t\text{DB}. \\ \sigma\text{DC} . \sigma\text{DB} &:: \sigma\text{AC} . \sigma\text{AB}. \\ \text{DB} \perp \text{AB} &= \text{AD}. \end{aligned}$$

$$\begin{aligned} \text{VI. } {}^t\text{DC.R} &:: \sigma\text{D} . {}^t\text{DB}. \\ \text{DA} \curvearrowright \text{DB} &= \text{AB}. \\ \sigma\text{DB} . \sigma\text{AB} &:: \sigma\text{DC} . \sigma\text{AC}. \end{aligned}$$

$$\begin{aligned} \text{VII. } {}^t\text{AC.R} &:: \sigma\text{C} . {}^t\text{CB}. \\ {}^t\text{D} . {}^t\text{C} &:: {}^s\text{CB} . {}^s\text{DB}. \\ \text{CB} \perp \text{DB} &= \text{CD}. \end{aligned}$$

$$\begin{aligned} \text{VIII. } {}^t\text{AC.R} &:: \sigma\text{C} . {}^t\text{CB}. \\ \text{CD} \curvearrowright \text{CB} &= \text{DB}. \\ {}^s\text{DB} . {}^s\text{CB} &:: {}^t\text{C} . {}^t\text{D}. \end{aligned}$$

$$\begin{aligned} \text{IX. } {}^t\text{D.R} &:: \sigma\text{DC} . {}^t\text{DCB}. \\ \text{DCA} \curvearrowright \text{DCB} &= \text{ACB}. \\ {}^s\text{DCB} . {}^s\text{ACB} &:: \sigma\text{D} . \sigma\text{A}. \end{aligned}$$

$$\begin{aligned} \text{X. } {}^t\text{D.R} &:: \sigma\text{DC} . {}^t\text{DCB}. \\ \sigma\text{D} . \sigma\text{A} &:: {}^s\text{DCB} . {}^s\text{ACB}. \\ \text{DCB} \perp \text{ACB} &= \text{DCA}. \end{aligned}$$

$$\text{XI. } {}^t\frac{1}{2}\text{DC}.$$

$$\text{XI. } \frac{T_{\frac{1}{2}}^I CD \cdot T_{\frac{1}{2}}^I CA + AD}{T_{\frac{1}{2}}^I CA - AD \cdot T_{\frac{1}{2}}^I CB - DB} :$$

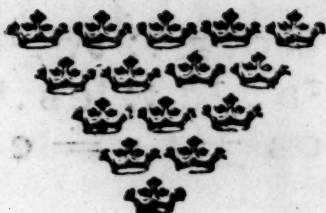
$$\frac{T_{\frac{1}{2}}^I CA - AD \cdot T_{\frac{1}{2}}^I CB - DB}{T_{\frac{1}{2}}^I CD + T_{\frac{1}{2}}^I CB - DB} = \frac{DB}{CB}.$$

$$\frac{T_{\frac{1}{2}}^I CD + T_{\frac{1}{2}}^I CB - DB}{S AD \cdot R :: S DB \cdot S DAB} = \frac{DB}{CB}.$$

$$S AD \cdot R :: S DB \cdot S DAB.$$

$$S AC \cdot R :: S CB \cdot S CAB.$$

$$DAB + CAB = DAC.$$





INTRODUCTION  
TO THE  
USE of both *GLOBES*.

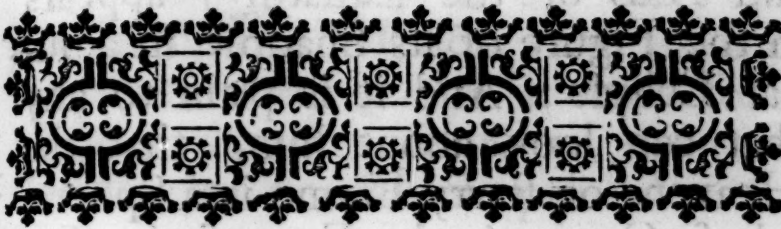




INTRODUCTION

TO THE

USE of both GLOBES.



# INTRODUCTION

T O T H E

U S E of both *GLOBES*.

**H**AVING already spoke of Circles on a Sphere in general, we shall now speak in particular of such Circles as are for ordinary marked on the artificial Globes; which Circles represent these imaginary ones on the natural Globe, with relation to which we calculate the Places, and apparent diurnal and annual Motions of the heavenly Bodies, and by Means of which we account for the Phenomena depending on these Motions. That this Earth is globular (abstracting from that very inconsiderable Difference of the Diameter

I 2

and

and Axis of the Equator ) appears in the Figure of its Shadow in a lunar Eclipse : And it is plain to any that can look about him, that the Heavens or starry Frame is a Sphere, whose Center is the Eye of the Beholder, or (to please the *Ptolemeans*, if there be any ) the Center of this Earth. For once then we shall conceive the Heavens and Earth to be Two concentric Spheres, and then whatever Circles we imagine in the Heavens, we may conceive Circles in the same Position with relation to each other on this Earth ; so the same Circles may be described both on the Celestial and Terrestrial Globes.

The more remarkable great Circles on the Sphere are these :

- |                     |                               |
|---------------------|-------------------------------|
| 1. <i>Equator,</i>  | 5. <i>Equinoctial Colure,</i> |
| 2. <i>Meridian,</i> | 6. <i>Solstitial Colure,</i>  |
| 3. <i>Ecliptic,</i> | 7. <i>Vertical Circles,</i>   |
| 4. <i>Horizon,</i>  | 8. <i>Hour Circles.</i>       |

I. The *Equator* is a great Circle passing from East to West, and having the same Poles and Axis with those of the World.

It



It is the Measure of the Sun's apparent diurnal Motion, for in the Space of a Day (*i. e.* Twenty four Hours) the Sun seems to describe this Circle, or a Circle parallel to it. Hence,

1.  $15^{\circ}$  of the Equator answer to an Hour of Time, and  $1^{\circ}$  to 4'. And therefore,

2. To reduce Degrees of the Equator to Time, divide by 15, and multiply the Remainder by 4, the Quote gives Hours, and the Product Minutes.

The same Way are Minutes of the Equator reduced to Minutes and Seconds of Time.

II. *Hour-Circles* are great Circles passing through the Poles of the World, and every  $15^{\circ}$  of the Equator.

*N. B.* The Point directly above our Head is called, the *Zenith*; and that directly below our Feet, the *Nadir*.

III, The *Meridian* is a great Circle passing through the Zenith and Nadir, and the Poles of the World.

It is a moveable Circle, for it shifts its Place as the Zenith does, so every Point in the Globe has its own Meridian;

dian; and therefore the artificial Globe is hung in a brazen Circle, which passes through the Poles of the Sphere, that any Point of the Sphere being brought to that Circle, it may represent the Meridian of that Point.

For the same Reason Geographers have fixt upon one Meridian, from which they reckon the Distances of all other Meridians, and this they call the *First Meridian*.

*Definitions for the Terrestrial Globe.*

IV. *Latitude* is the Distance of a Place from the Equator, and is either North or South.

V. *Longitude* is the Distance of the Meridian of any Place from the first Meridian. It is always reckoned Eastward.

VI. The *Distance* of Two Places is an Arch of a great Circle intercepted betwixt these Two Places.

The Brazen Meridian is divided into Four Quadrants, the Two upper Quadrants are divided into  $90^\circ$ , and reckoned from the Equator to the Poles;

Poles; the other Two Quadrants are reckoned from the Poles to the Equator.

On the Two upper Quadrants are reckoned the Latitudes of Places.

The Equator is divided into  $360^\circ$ , reckoned from the first Meridian Eastward completely round the Globe; on it are marked the Longitudes of Places.

VII. The *Ecliptic* is a great Circle cutting the Equator at Angles of  $23^\circ 30'$ ; it is the Way of the Sun's apparent annual Motion.

VIII. The *Ecliptic* is divided into Twelve equal Parts, called *Signs*, each Sign contains  $30^\circ$ , for  $\frac{360^\circ}{12} = 30^\circ$ .

The Sun describes this Circle  $360^\circ$  in the Space of 365 Days, and consequently he moves through almost  $1^\circ$  in one Day.

IX. The Time of the Sun's Revolution round the *Ecliptic* is called a *Year*.

X. The Year is divided, as the *Ecliptic* is, into Twelve Parts, almost equal, called *Months*.

The



The Signs, with their correspondent Months and Characters, stand thus:

<sup>♈</sup> Aries, <sup>♉</sup> Taurus, <sup>♊</sup> Gemini, <sup>♋</sup> Cancer, <sup>♌</sup> Leo,  
March, April, May, June, July,

<sup>♍</sup> Virgo, <sup>♎</sup> Libra, <sup>♏</sup> Scorpio, <sup>♐</sup> Sagittary,  
August, September, October, November,

<sup>♑</sup> Capricorn, <sup>♒</sup> Aquary, <sup>♓</sup> Pisces.  
December, January, February.

XI. The *Horizon* is that Circle described by our Eye, when placed on a high Mountain, or when at Sea, we think we see the Heavens and Earth joyn, and this is called the *Sensible Horizon*.

XII. A great Circle parallel to this is called the *Rational Horizon*, its Poles are the Zenith and Nadir.

The rational Horizon is represented by the Wooden Horizon on the artificial Globe. On it are marked,

1. A Circle divided into Signs and Degrees, as the Ecliptick is, beginning with the first Degree of  $\gamma$  at the East Point of the Horizon, and reckoning Northward.

2. A double Kalendar, viz. the *Julian* and *Gregorian*, each divided into Months and Days answering to the Divisions of the former Circle, beginning the *Julian* Kalendar with *March* 11. at the East Point of the Horizon, and the *Gregorian* with *March* 22. at the same Point, and reckoning both Northward.

3. A Circle divided, as the Sea Plate III.  
Compass is, into Thirty two equal FIG. 26.  
Parts, called *Rumbs*, each Rumb contains  $11^{\circ}. 15'$ . for  $\frac{360^{\circ}}{32} = 11^{\circ}. 15'$ .

XIII. The Elevation of the Pole is its Altitude above the Horizon; it is measured on the lower Quadrants of the brazen Meridian.

The Elevation of the Pole is equal to the Latitude of the Place.

For making  $z$  the Place, whose Ho- FIG. 27.  
rizon is  $HR$ ,  $QE$  the Equator, and  $p$ ,  
 $p$  the Poles,  $Qz$  is the Latitude,  $^a 4 b$ .  
 $PR$  the Elevation of the Pole, and it  
K is

<sup>b</sup> 2. Trig.  
P. IV.

is plain  $zR^b = 90^\circ{}^b = PQ$  and  $zR - PZ = PQ - PZ$ . *i. e.*  $PR = QZ$ .  $\mathcal{Q}$ .  
*E. D.*

When the Sun is on the Equator (*i. e.* at either Point where the Ecliptick cuts the Equator) the Days and Nights are equal all the World over.

For it is plain  $QO = OE$ , that is,  $\frac{1}{2}$  the Equator is in Darkness, the other in Light. Hence the  $1^\circ$  of  $\gamma$  and  $\alpha$  are called the *Equinoctial Points*.

Plate IV.  
FIG. 28.

XIV. A *Right Sphere* is, when the Equator is at right Angles with the Horizon.

This Position of the Sphere they have that live at the Equator. The Poles of the World coincide with the North and South Points of their Horizon, and their Days and Night are equal all the Year over: For the Equator and all its Parallels are bisected by the Horizon.

FIG. 29.

XV. A *Parallel Sphere* is, when the Equator  $EQ$  coincides with the Horizon  $HR$ , and consequently all its Parallels are parallel to the Horizon.

This



Plate. 3c

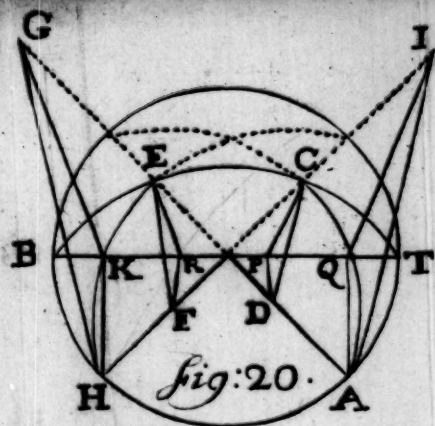


fig: 20.

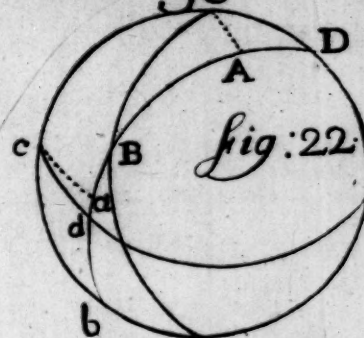


fig: 22.

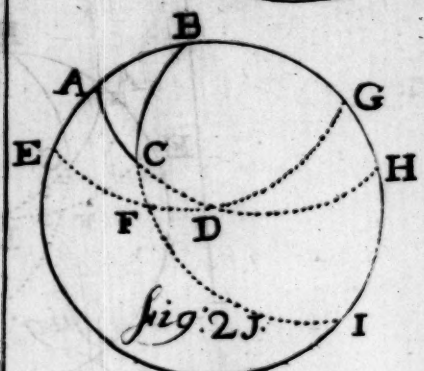


fig: 23.

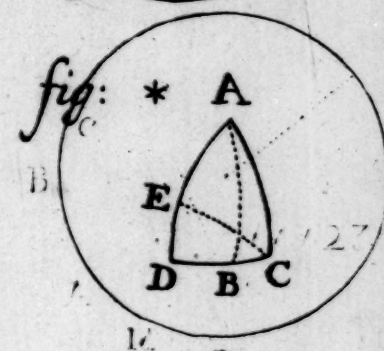


fig: \*

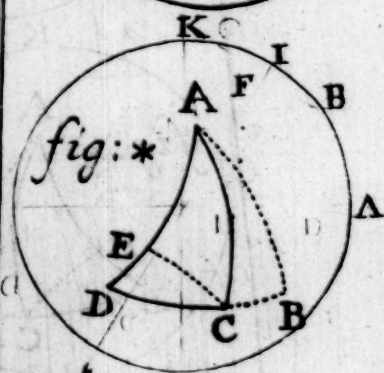


fig: \*

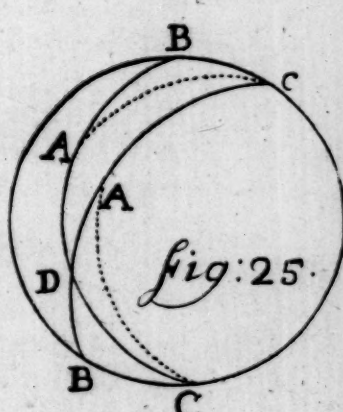


fig: 25.

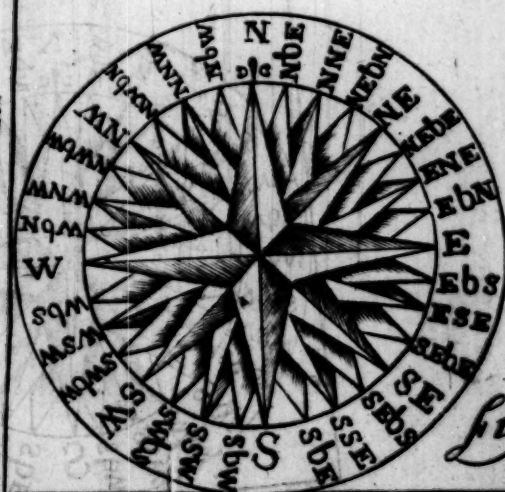


fig: 26.

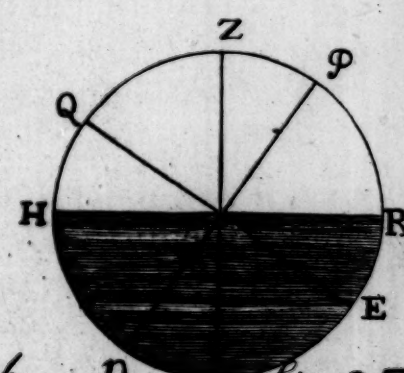


fig: 27.

This Position of the Sphere is proper to those that live at the Poles, and so the Poles of the World coincide with their Zenith and Nadir. Seeing the Sun can at once illuminate but Half the Globe, supposing him on the Equator at *E*, he then shines precisely to both Poles, and the Equator *E Q*, and all its Parallels, *r* *Q*, *t* *v*, &c. have just one Half in Light, and the other Half in Darkness, that is, the Days and Nights are every where equal.

Supposing now the Sun at *x*, viz.  $10^{\circ}$  North from the Equator, he shall then shine to *v*,  $10^{\circ}$  beyond *p*, and to *v*,  $10^{\circ}$  short of *p*; and describing a Circle upon *p*, as a Pole parallel to the Equator, and passing through *v*, the whole Tract lying within that Circle shall have the Sun continually in View, and consequently they shall have continued Day. It is plain, that the Tract lying within the like Parallel, passing through *v*, shall have continued Darkness.

In like Manner the Sun being at *r*, the Point of his greatest Declination

Northward from the Equator, *viz.*  $23^{\circ}. 30'$ . he shall shine to *c*,  $23^{\circ}. 30'$  Southward from the Pole *p*, and to *A*,  $23^{\circ}. 30'$  short of *p*; and all that Tract lying within the Parallel *A c*, hath the Sun constantly in View, &c.

Thus from the Time of the Sun's being on the Equator at *e*, to the Time of his greatest Declination at *r*, the Sun has been continually in View of *p*, and *p* has been still in Darknefs, that is, for the Space of Three Months, answering to Three Signs,  $\gamma$ ,  $\delta$ ,  $\Pi$ , for the first Degree of  $\mathfrak{z}$  is the Point of the Sun's greatest Declination Northward.

The Pole *p* has the Sun still in View during the following Three Months, while he describes the Three Signs,  $\mathfrak{z}$ ,  $\alpha$ ,  $\mu$ , for all this while he is on the North-side of the Equator, and so shines beyond its North Pole. So the Inhabitants of the North Pole, if there are any, have had Half a Year's Day, and those of the South Pole Half a Year's Night, *viz.* from *March 10 to September 13*.

The Sun, at  $1^{\circ} \approx$ , is again on the Equator, and the Days and Nights are again



again equal; thence he declines from the Equator Southward, and leaves the North Pole in continued Darknes, while he gives Half a Year's Light to the South Pole.

The Point of the Sun's greatest Declination Southward is the  $1^{\circ}$  of  $\varpi$ .

XVI. The Two Points of greatest Declination, are called the *Solstitial Points*, and we having North Latitude,  $1^{\circ}$   $\odot$  is called our *Summer Solstice*, and  $1^{\circ}$   $\varpi$  our *Winter Solstice*.

XVII. A great Circle passing thro' the Poles of the World and the Solstitial Points, is called the *Solstitial Colure*.

XVIII. A great Circle drawn thro' the Poles of the World and Equinoctial Points, is called the *Equinoctial Colure*.

XIX. A Circle drawn thro'  $1^{\circ}$   $\odot$ , and parallel to the Equator, is called the *Tropick of Cancer*, for the Sun at that Point begins to return to the Equator.

XX. A Parallel to the Equator, drawn thro'  $1^{\circ}$   $\varpi$ , is called the *Tropick of Capricorn*.

XXI. An

FIG. 30.

XXI. An *Oblique Sphere* is, when the Equator  $EQ$ , and all its Parallels, make oblique Angles with the Horizon  $HR$ , or  $hr$ .

This Position of the Sphere they have that live betwixt the Equator and the Poles. The Pole nearest to the Zenith is always elevated above the Horizon, and the other depressed below it. The Elevation or Depression of the Pole is always equal to the Latitude of the Place. Hence,

The Distance between the Poles of the World, and those of the Horizon, is equal to the Co-latitude of the Place; likewise the Elevation of the Equator above the Horizon on one Side, and its Depression below it on the other, is equal to the Co-latitude.

1. Let the Sun be on the Equator at  $E$ , whatever the Horizon be, whether  $HR$ , or  $hr$ , still the semidiurnal Arch  $EO = 90^\circ = OQ$  the seminocturnal Arch, that is, the Sun being on the Equator, the Days and Nights are equal all the World over. Make now the Horizon  $HR$ .

2. Sup-

2. Supposing the Sun at  $b$ , then is the semidiurnal Arch  $bu \subset by$ , but  $by$  and  $eo$  are similar Arches, then the Day, when the Sun is at  $b$ , is longer than when at  $e$ . Again, the Sun being at  $r$  the solstitial Point, the semidiurnal Arch  $tm \subset tv$ , *i. e.*  $tm \subset bu$ , *i. e.* the Days increase as the Sun approaches nearer to the Zenith, & *e contra*.

It is plain  $tm$  is the greatest semidiurnal Arch, and consequently the Days are at the longest when the Sun is on the solstitial Point next to the Zenith.

3. The greater the Latitude is, the greater the longest Day is.

Make the Horizon  $hr$ , where  $pr$  the Elevation of the Pole, equal to the Latitude, is greater than  $pr$ , the Elevation of the Pole above the Horizon  $hr$ , it is plain the semidiurnal Arch  $tm \subset tm$ .

4. If the Latitude  $ez = 66^\circ. 30'$ . FIG. 31. the Complement of the Sun's greatest Declination, their longest Day shall be Twenty four Hours, for the Elevation of the Equator  $eh = \text{Co-lat.} = 23^\circ. 30'$ .  
the



the Sun's greatest Declination, therefore the Horizon  $HR$  coincides with the Ecliptick, and the semidiurnal Arch  $TR$  coincides with the Tropick of Cancer, and so touches, but does not cut the Horizon.

The Poles of the Meridian are the East and West Points of the Horizon. Hence, the Horizon is a moveable Circle; for the Meridian being moveable, its Poles are so too, and with them the Horizon moves.

For this Reason the Globe is so set in the wooden Horizon, that any Point of the Globe may be brought to the Vertex or Zenith, and so it represents the Horizon of that Place.

The Poles of the Equinoctial Colure are the Two Points where the solstitial Colure cuts the Equator, & *vicissim*.

The Poles of the Ecliptick are Two Points in the solstitial Colure, as far distant from the Poles of the Equator, as the Ecliptick declines from the Equator.

XXII. Two Circles parallel to the Equator, described by one Revolution of  
of

*Use of both Globes.* 81

of the Poles of the Ecliptick round the Globe, are called *Polar Circles*. That described by the North Pole of the Ecliptick, is called the *Arctic*, the other the *Antarctic* Polar Circle, as *AC*, *ac*.

The Two Tropicks, with the Two Polars, divide the Globe into Five *Zones*.

XXIII. That lying betwixt the Two Tropicks, is called the *Torrid Zone*, as being exposed to the direct Stroke of the Sun's Rays, and consequently to the most violent Heats.

XXIV. The Two Zones lying within the Polars, are called the *Frigid Zones*, the Stroke of the Sun's Rays being there very oblique, and consequently faint and weak.

XXV. The Two Zones lying betwixt the Tropicks and Polars, are called *Temperate Zones*, as partaking equally of the Heats of the Torrid, and Colds of the Frigid Zones.

The Inhabitants of the several Zones take their Names from the various Projections of their Shadows, when the Sun is on their Meridians.

L

XXVI. The

XXVI. The Inhabitants of the Torrid Zone are called *Amphiscii*, as having their Shadows projected sometimes North, sometimes South, according as the Sun is on the South or North-side of their Zenith.

To these the Sun is vertical twice a Year, and then they are *Ascii*, having no Meridian Shadow.

To these that live under the Tropic, the Sun is vertical but once a Year.

XXVII. These in the Frigid Zones are called *Periscii*; for they having the Sun still in View, their Shadows describe a Circle once in Twenty four Hours.

XXVIII. The Inhabitants of the Temperate Zones are called *Heteroscii*, as having their Shadows projected only one Way.

Supposing Parallels of Latitude to be drawn through every Degree and Minute of the Meridian, which we shall call simply *Parallels*, and calling the Two opposite Semicircles of the same Meridian, as they are cut off by the Axis of the World, *Opposite Meridians*,



*ridians*, we may then divide the Inhabitants of the Globe into *Antæci*, *Periæci* and *Antipodes*.

XXIX. *Antæci* have the same Meridian, but opposite Parallels; and consequently they have the same Days and Nights, but opposite Seasons, and opposite Poles elevated.

XXX. *Periæci* have the same Parallel, but opposite Meridians: They have the same Seasons, and the same Pole elevated, but opposite Days and Nights.

XXXI. *Antipodes* have opposite Parallels, and opposite Meridians: They have opposite Seasons, opposite Days and Nights, and opposite Poles elevated.

XXXII. *Climates* are little Zones terminated by Two Parallels of Latitude, so taken, as the longest Day at the greater Latitude is Half an Hour more than the longest Day at the lesser.

The longest Day at the Equator is Twelve Hours, and at either Polar it is Twenty four Hours. The Number of *Climates* then betwixt the Equator

and either Polar is Twenty four, and their Number betwixt the Two Polars is Forty eight.

The longest Day at either Pole is Six Months, and allowing the Days to increase by Months from the Polars to the Poles, there shall then be Six Climates from each Polar to its nearest Pole, in both Twelve, which, added to the former Forty eight, give in all Sixty Climates.

If the longest Day be supposed to increase by a Quarter of an Hour, these *Zonulae* are called *Parallels*.

Because the right and left Parts of Heaven are frequently mentioned, we shall here observe, that

1. Philosophers and Geographers called the East the right, and the West the left Parts. The former, because the heavenly Bodies apparently moved from East to West : The latter, because, in observing the Pole's Altitude, the East was on their right Hand.

2. Astronomers reckoned West the right, and East the left Parts : For they, looking Southward in observing the

the Motions of the Stars, had the West on their right, and East on their left Hand.

3. Augurs, looking toward the rising Sun, reckoned South the right, and North the left Parts.

4. Poets, looking towards the setting Sun, reckoned North the right, and South the left Parts.

Hence, OVID. *Met. Lib. 2.*

*Neu te dexterior tortum declinet ad anguem,  
Neve sinisterior pressam rota ducat ad aram.*

Here too, having formerly mentioned both Kalendars, we shall shew how to find the *Golden Number*, *Epaet*, and the *Dominical Letter* for the *Julian* Kalendar.

XXXIII. The *Golden Number* is the Space of Nineteen Years, the Time of one Revolution of the Moon's *Nodes* (*i. e.* the Two Points where her Orbit cuts the *Ecliptick*) round the *Ecliptick*.

XXXIV. The *Epaet* is the Space of Eleven Days, the Excess of the solar above the lunar Year; the former containing 365. and the latter 354 Days.

*To*



To find the GOLDEN NUMBER and  
EPACT for any Year.

To the Year given add Unity, divide that Sum by 19. the Remainder is the *Golden Number*.

$$\begin{array}{r}
 1713 \\
 + 1 \\
 \hline
 19) 1714 (90 \\
 \quad 4 \text{ G. N.} \\
 \quad 11 \\
 \hline
 30) 44 (1 \\
 \quad 30 \\
 \hline
 \quad 14 \text{ Ep.} \\
 \hline
 \end{array}$$

Multiply the *Golden Number* by 11. divide the Product by 30. the Remainder is the *Epaet*.

So 4 is the *Golden Number*, and 14 the *Epaet* for the Year 1713.

To the *Epaet* add the Day of the Month, and Number of Months from *March inclusive*; that Sum (if less than 30)

$$\begin{array}{r}
 \text{Epaet} \quad 14 \\
 \text{July} \quad 5 \\
 \text{Day} \quad 14 \\
 \hline
 \quad 33 \\
 \quad - 30 \\
 \hline
 \text{Moon's Age, } 3
 \end{array}$$

is the Moon's Age, if more, the Excess is the Moon's Age.

So 3 is the Moon's Age, July 14. 1713.

To find the Change of the Moon for any Month.

To the *Epaēt* add the Number of Months from *March* *inclusivè* ; subtract that Sum from 30. or, if more than 30. from 60. the Remainder is the Time of Change.

$$\begin{array}{r} \text{Epaēt} \quad 14 \\ \text{August} \quad 6 \\ \hline \quad \quad 20 \\ \hline 30 - 20 = 10 \end{array}$$

So the Moon changes, *August* 10. 1713.

N. B. The *Epaēt* for any Year does not reckon till *March*.

To find the Time of the Moon's coming to the Meridian.

Multiply the Moon's Age by 4. divide the Product by 5. the Quote gives Hours, the Remainder multiplied by 12 gives Minutes.

For 12° being the Moon's mean Motion from the Sun in one Day, that multiplied

$$\begin{array}{r} 11 \\ 4 \\ \hline 5 \overline{) 44} ( 8^h. 48' \\ 40 \\ \hline 4 \\ 12 \\ \hline 48 \end{array}$$

by

by 4 (*viz.*  $4 \times 12^\circ$ ) gives the same Arch in Minutes of Time; and making  $x$  the Moon's Age, or Number of Days from the Conjunction, at which Time the Sun and Moon were at once on the Meridian,  $4 \times 12 \times x$  shall give the Moon's Distance from the Sun in Minutes of Time, which, divided by 60 ( $= 5 \times 12$ ) gives  $\frac{4x \times 12}{5 \times 12} = \frac{4x}{5}$  the same Distance in Hours, and what remains shall be Fifth Parts of an Hour, *i. e.*  $12'$ .

So on the 11 Day of the Moon's Age she is on the Meridian at  $8^h.48'$ . Hence,

- I. *To find the Time of high Water at any Port, the Time of high Water at the full or Change being given.*

The Sum of the Time of the Moon's Southing, and that of high Water at the Change, gives the Answer.

So at *London* high Water at the Change is at 3. therefore at the 11 Day of the Moon it is  $3^h + 8^h.48' = 11^h.48'$ .

II. *To*



II. To find the Hour of the Night by the Moon shining on a Sun-dial.

To the Time of the Moon's South-ing, add the Distance of the Hour on the Dial from 12. if the Shadow falls amongst the Afternoon Hours, if not, subtract.

XXXV. In the Kalendar is a continued Revolution of the first Seven Letters of the Alphabet, representing the Seven Days of the Week ; by these we may find the *Feria*, or Day of the Week answering to any Day of the Year, provided we know which of these Letters represents *Sunday*, or is the *Dominical* or *Sunday Letter* for that Year.

To find the DOMINICAL LETTER for any Year.

Divide the Year given by 4. ( the Remainder gives the Year after Leap-year, and if 0 remain, it is Leap-year ) to the Dividend add the Quote, and the Number 5. divide that Sum by 7. sub-  
M tract

$$\begin{array}{r} 4) 1713 \\ \underline{428} \end{array} \left. \vphantom{\begin{array}{r} 4) 1713 \\ \underline{428} \end{array}} \right\} 1 \text{ remains.}$$

$$\begin{array}{r} 5 \\ \hline \end{array}$$

$$\begin{array}{r} 7) 2146 \\ \underline{306} \end{array} \left. \vphantom{\begin{array}{r} 7) 2146 \\ \underline{306} \end{array}} \right\} 4 \text{ remains.}$$

$$\begin{array}{r} 8 - 4 = 4 \\ \hline \end{array}$$

|    |    |    |    |
|----|----|----|----|
| I. | 2. | 3. | 4. |
| A. | B. | C. | D. |

tract the Remainder from 8. what remains gives the Number of Letters from A. So D is the *Dominical Letter* for the Year 1713. being the first after Leap-year.

N. B. Because of the Intercalation of a Day in Leap-year, at the 6 *Cal. Mart* (i. e. February 24) Leap-year has Two Letters, the first reckoning from January 1. to February 24. the other for all the rest of the Year.

*To find the DOMINICAL LETTERS for Leap-year.*

Find the Letter as if it were a

$$\begin{array}{r} 4) 1720 \\ \underline{430} \end{array} \left. \vphantom{\begin{array}{r} 4) 1720 \\ \underline{430} \end{array}} \right\} 0 \text{ remains.}$$

$$\begin{array}{r} 5 \\ \hline \end{array}$$

$$\begin{array}{r} 7) 2155 \\ \underline{307} \end{array} \left. \vphantom{\begin{array}{r} 7) 2155 \\ \underline{307} \end{array}} \right\} 6 \text{ remains.}$$

$$\begin{array}{r} 8 - 6 = 2 \\ \hline \end{array}$$

|    |    |
|----|----|
| I. | 2. |
| A. | B. |

common Year, and prefix to it the Letter immediately after it in the natural Order of the Alphabet.

So CB are the *Dominical Letters* for the

Year 1720. being Leap-year.

The

# Use of both Globes.

91

The Letters answering to the first Day of every Month in the Kalendar, are the same with the first Letters of the Words in this Distich.

*Astra, Dabit, Dominus, Gratisfq; Beabit, Egenos,*  
Jan<sup>ry</sup>. Feb<sup>ry</sup>. March, April, May, June,  
*Gratia, Christicolæ, Feret, Aurea, Dona, Fideli.*  
July, August, Sept<sup>r</sup>. Oct<sup>r</sup>. Nov<sup>r</sup>. Dec<sup>r</sup>.

*Given the Dominical Letter, to find what Day of the Week any Day of any Month is.*

Find the Day of the Week answering to the Letter of the first Day of the Month, that Day shall be the 1, 8, 15, 22, 29 Days of that Month; the other Days are easily found.

The same may be done by this Kalendar. Thus,

Find the Month on the Top, the

| January<br>October | May    | August  | Feb. March<br>November | June     | September<br>December | April<br>July |
|--------------------|--------|---------|------------------------|----------|-----------------------|---------------|
| Sunday             | Monday | Tuesday | Wednesday              | Thursday | Friday                | Saturday      |
| A                  | B      | C       | D                      | E        | F                     | G             |
| B                  | C      | D       | E                      | F        | G                     | A             |
| C                  | D      | E       | F                      | G        | A                     | B             |
| D                  | E      | F       | G                      | A        | B                     | C             |
| E                  | F      | G       | A                      | B        | C                     | D             |
| F                  | G      | A       | B                      | C        | D                     | E             |
| G                  | A      | B       | C                      | D        | E                     | F             |
| 1                  | 2      | 3       | 4                      | 5        | 6                     | 7             |
| 8                  | 9      | 10      | 11                     | 12       | 13                    | 14            |
| 15                 | 16     | 17      | 18                     | 19       | 20                    | 21            |
| 22                 | 23     | 24      | 25                     | 26       | 27                    | 28            |
| 29                 | 30     | 31      |                        |          |                       |               |

M 2

Let-



Letter immediately below it answers to the first Day of that Month: Find the *Dominical Letter* on the left Hand, the Day of the Week above the first Day's Letter gives the first Day of the Month. Find the first Day's Letter on the left Hand, the Letter opposite to it, and directly above the given Day of the Month, is the Letter for that Day.

The preceeding Problems relate to the *Julian* Kalendar, where the Year consists of 365 Days, 6 Hours; the common Year consisting of 365 Days, and the odd 6 Hours making 24 Hours, or one Day in Four Years, that Day is intercaled in *February* every Fourth Year, so that every Fourth Year consists of 366 Days, and is called *Bissextile*, or *Leap-year*.

The Solar tropical Year, consisting of 365 Days, 5<sup>h</sup>. 49'. this *Julian* Year exceeds it by 11' *fere*; so that the Times of the Equinoxes shift backwards almost 11' every Year, and consequently one Day in about 133 Years.

Pope *Gregory XIII.* observing, that from the Time of the *Nicene* Council,

cil, *Anno Domini* 325. to his Time, *Anno Domini* 1582. the Times of the Equinoxes had shifted Ten Days, and with them the Terms of the moveable Feasts of the Church had likewise shifted, ordered that, in 1582. the 5 of *October* should be reckoned the 15 of that Month, and, by this Means, the 11 of *March*, on which the vernal Equinox fell out, according to the common Course of the Years in his Time, became the 21 of *March*, upon which Day the Equinox happened at the Time of the *Nicene* Council. Thus the Times of the Equinoxes, and with them the Terms of the moveable Feasts, were restored to their ancient Situation.

*Next*, To prevent the like Relapse of the Equinoxes, he appointed that every Hundredth Year, which, in the *Julian* Stile, is always Leap-year, should be reckoned a common Year; and, by this Means, added, every Century, one Day to the Difference of the *Julian* and *Gregorian* Stiles. Thus, from 1600 to 1700. the Difference of the Two Stiles, was Ten Days,

94      *Introduction to the*

Days, from 1700 to 1800. it is Eleven Days, &c. Hence,

I. *To reduce the JULIAN Stile to the GREGORIAN.*

Add the Difference of Stiles to the Day of the Month, *Julian* or Old Stile, and you have the Day of the Month in the *Gregorian* or New Stile.

II. *To reduce the GREGORIAN to the JULIAN Stile.*

Subtract the Difference of Stiles from the Day of the Month New Stile, and you have the Day of the Month Old Stile.

RULE I. *To find the GOLDEN NUMBER New Stile.*

It is done as before in Old Stile.

II. *To find the EPOCH New Stile.*

From the *Epoch* Old Stile (augmented by 30. if Need be) subtract the

Dif-



Difference of the Stiles, and you have the *Epaet* New Stile.

III. *To find the DOMINICAL LETTER  
New Stile.*

From the absolute Number of *Dominical Letters*, from the Beginning of the Period of Letters current at our Saviour's Incarnation, subtract the Difference of Stiles, divide the Remainder by 7. what remains gives the Letter from G, or that Number subtracted from 8. gives the Letter from A.

XXXVI. *Vertical Circles* are great Circles passing through the Zenith and Nadir, and any Point of the Globe taken at Pleasure.

They are represented by the Quadrant of Altitude, which is a brazen Quadrant that is screwed to the Zenith, and is moveable completely round the Globe.

XXXVII. Circles passing through the Poles of any great Circle, are called *Secondaries* to that Circle.

So

So Verticals are Secondaries to the Horizon, and Meridians are Secondaries to the Equator.

PROB. I.

*TO rectify the Globe for the Latitude of any Place.*

Elevate the Pole to the Latitude of the Place, and screw the Quadrant of Altitude to the Zenith.

PROBLEMS *on the Terrestrial Globe.*

PROB. II.

*TO find the Longitude and Latitude of a given Place.*

Bring the Place to the brazen Meridian, the Degree of the Meridian above the Place is its Latitude, the Degree of the Equator cut off by the Meridian, is its Longitude.

PROB. III.

*Given the Longitude and Latitude of a Place, to find the Place on the Globe.*

Bring

Bring the Longitude to the brazen Meridian, the Place is below the Latitude.

PROB. IV.

*TO find the Distance of Two Places.*

Rectify the Globe for the Latitude of one of these Places, bring the Quadrant of Altitude to the other, the Arch intercepted betwixt the Two Places gives the Distance in Degrees, which, multiplied by 60. gives the Distance in Geometrick Miles.

*Definitions for the Celestial Globe.*

XXXVIII. *Longitude* is the Distance of the Sun or Star from the first Degree of  $\gamma$ , reckoned on the Ecliptick.

XXXIX. *Latitude* is the Star's Distance from the Ecliptick reckoned on the Secondaries to the Ecliptick, it is either North or South.

N. B. The Sun has no Latitude.

N

XL. De-



XL. *Declination* is the Distance of the Sun or Star from the Equator, reckoned on the Secondaries to the Equator. It is either North or South.

XLI. *Right Ascension* is the Degree of the Equator, cut off by the Horizon, when the Sun or Star is brought to the Horizon in a right Sphere, or because in all Positions of the Sphere the Equator is still perpendicular to the Meridian.

Right Ascension is the Degree of the Equator, cut off by the brazen Meridian, when the Sun or Star is brought to that Meridian in any Position of the Sphere.

XLII. *Oblique Ascension* is the Degree of the Equator, cut off by the Horizon, when the Sun or Star is brought to the Horizon in an oblique Sphere.

XLIII. *Ascensional Difference* is the Difference betwixt the right and oblique Ascension.

N. B. If the Latitude of the Place, and Declination of the Sun or Star be towards the same Part, viz. both North, or both South, the right Ascension is the greater, & *e contra*.

XLIV. The

XLIV. The Sun or any Star is said to rise, when it comes to the East-side of the Horizon, and to set, when it comes to the West-side of the Horizon.

The Poets have a threefold Rising of the Stars, viz. *Cosmick, Achronick* and *Heliack*.

XLV. A Star is said to rise cosmically, when it rises with the Sun. So VIRG. *Geo. Lib. I.*

*Candidus auratis aperit cum cornibus annum  
Taurus, & adverso cedens canis occidit astro.*

i. e. When the Sun enters *Taurus*, and they both rise at once.

A Star sets cosmically when it sets at Sun-rising, or rises at Sun-setting. So VIRG. *Geo. Lib. I.*

*Ante tibi eoa Atlantides abscondantur,  
Debita quam sulcis committas semina, quamque  
Invitæ properes anni spem credere terræ.*

Harvest-time is meant, when the Sun being in *Scorpio*, the Pleiades set cosmically.

XLVI. A Star is said to rise achronically, when it rises at Sun-setting.

*Ut careo vobis Stygias detrusus in oras,  
Quattuor autumnos Pleias orta facit.* OVID.

The Sun being in *Scorpio*, the *Pleiades* rise achronically, and set cosmically.

A Star is said to set achronically, when it sets with the Sun. So OVID. *Fast. Lib. 2.*

*Quem modo cœlatum stellis delphina videbas,  
Is fugiet visus nocte sequente tuos.*

*February 3.* when the Sun being in *Aquary*, the *Dolphin* sets achronically.

Hence, the Sign in which the Sun is, rises cosmically, and sets achronically, and the opposite Sign, at Night rises achronically, and in the Morning sets cosmically.

XLVII. The heliack Rising of a Star is, when the Star appears in View after having been for some Time lost in the Sun's greater Light. So OVID. *Fast. Lib. 2.*

*Jam levis obliqua subsedit aquarius urna,  
Proximus athereos excipe piscis equos.*

About the latter End of *February*,  
*Ante tibi eœ Atlantides abscondantur,  
Gnosiaque ardentis decedat stella coronæ,  
Debita quam sulcis committas semina, &c.*

VIRG. *Geo. 1.*  
The



The Sun being in *Scorpio*, the Pleiades set cosmically, and the *Corona Septentrionalis* rises heliacally.

XLVIII. The heliack Setting of a Star is, when the Sun's greater Light obscures that Star.

*Candidus auratis aperit cum cornibus annum  
Taurus, & adverso cedens canis occidit astro.*

XLIX. *Amplitude* is the Distance of the rising or setting Sun or Star, from the East or West Point of the Horizon. It is either *Ortive*, when the Star rises, or *Occidual*, when it sets.

L. *Altitude* is the Elevation of the Sun or Star above the Horizon, reckoned on the Secondaries to the Horizon.

LI. *Azimuth* is the Distance betwixt the North Point of the Horizon, and that Point where a Vertical, passing through the Body of the Sun or Star, cuts the Horizon.

Hence, Verticals are called *Azimuths*, or *Azimuth Circles*.

PROB.

## PROB. V.

**T**O find the Sun's Place on the Ecliptick at any Time.

In the Kalendar on the wooden Horizon find the Day of the Month, and opposite to it, on the Circle divided as the Ecliptick, you shall find the Sign and Degree the Sun is in. Find that Sign and Degree on the Ecliptick on the Globe, so you have the Sun's Place on the Globe. So *April* 3. the Sun's Place is  $\gamma$   $20^{\circ}.30'$ .

## PROB. VI.

**T**O find the Sun's Declination and right Ascension.

Bring the Sun's Place to the brazen Meridian, the Degree of the Meridian above the Sun's Place, gives his Declination; the Degree of the Equator cut off by the Meridian, gives his right Ascension.

*N. B.* The Co-latitude of any Place is equal to the Sum or Difference of the Declination and Meridian Altitude, according as the Latitude and Declination are toward, contrary, or the same Parts.

PROB.

PROB. VII.

**T**O find the Sun's ortive or occidual Amplitude, oblique Ascension, and ascensional Difference.

Rectify the Globe to the Latitude of the Place, and bring the Sun's Place to the East-side of the Horizon, the Distance betwixt the Sun's Place and the East Point of the Horizon, gives his ortive or occidual Amplitude, the Degree of the Equator cut off by the Horizon, is his oblique Ascension; so the ascensional Difference is easily found.

PROB. VIII.

**T**O find the Hour and Minute of the Sun's Rising and Setting, and the Length of the Day and Night.

Reduce the ascensional Difference to Time. So  $6^h \mp$  that Time (according as the Latitude and Declination are towards the same or contrary Parts) gives the Time of the Sun's Rising, and  $6^h \pm$  that Time gives his Set-



Setting: Double the Time of his Rising gives the Length of the Night, and Double the Time of his Setting gives the Length of the Day. Or thus,

Bring the Sun's Place to the brazen Meridian, fasten the horary Circle on that Meridian, with the Index pointing 12 of the Day, turn the Globe until the Sun is on the East-side of the Horizon, the Index shall point the Hour of Sun-rising, &c.

#### PROB. IX.

*TO find the Beginning of the Morning, and Ending of the Evening Twilight.*

Bring the Point of the Ecliptick, opposite to the Sun's Place, to the West-side of the Horizon, move the Globe and Quadrant of Altitude at once, till that Point be elevated  $18^{\circ}$  above the Horizon; the Difference betwixt the Point of the Equator on the East Point of the Horizon, and the oblique Ascension reduced to Time, and subtracted from the Time  
of

of Sun-rising, gives the Beginning of the Morning Twilight: In like Manner may the End of the Evening Twilight be found.

PROB. X.

**TO** find the Sun's Altitude and Azimuth at any Time of the Day; or, his Altitude given, to find his Azimuth, and the Hour of the Day.

1. Reduce the Hour from 12 given, to Degrees of the Equator, bring the Sun's Place to the brazen Meridian, and turn round the Globe, until that Arch of the Equator pass the Meridian; so the Quadrant of Altitude, laid on the Sun's Place, shall cut off his Azimuth on the Horizon, and shew his Altitude above his Place.

2. Bring the Sun's Place to the brazen Meridian, move the Globe and Quadrant of Altitude at once, until the Sun's Place be on the given Altitude, the Difference betwixt the right Ascension, and the Degree of the Equator cut off by the Meridian, gives  
O the

the Hour from 12 ; and the Quadrant cuts off the Azimuth on the Horizon.

The Two last Problems may be done by Help of the horary Circle.

The Declination, right and oblique Ascensions, Amplitude, Altitude and Azimuth of such fixt Stars, as rise and set, are found after the same Manner.

### PROB. XI.

*TO find the Longitude and Latitude of the fixt Stars.*

Elevate the Pole to the Altitude of  $66^{\circ}. 30'$ . bring the Pole of the Ecliptick to the brazen Meridian, screw the Quadrant of Altitude to the Zenith, and bring it to the Star proposed, the Degree of the Quadrant above the Star, gives its Latitude, and the Arch of the Ecliptick, cut off by the Quadrant, gives its Longitude.

*N. B.* All Stars, whose Declination is towards the same Parts with the Latitude of the Place, and less than its Colatitude, rise and set, otherwise they never set.

PROB.



PROB. XII.

*TO find the Continuation of any fixt Star above the Horizon.*

$6^h \pm$  the Star's ascensional Difference, according as its Declination is towards the same or contrary Parts with the Latitude of the Place, gives  $\frac{1}{2}$  its Continuation above the Horizon.

PROB. XIII.

*Given the right Ascensions of the Sun and any fixt Star, to find the Time of that Star's Culminating, i. e. of its Coming to the Meridian.*

The fixt Star's right Ascension ( $+ 360^\circ$ . if Need be ) — the Sun's right Ascension, gives the Distance from 12 of the Day to the Time of culminating.

PROB. XIV.

*Given the Time of culminating of any Star, and  $\frac{1}{2}$  its Continuation above the Horizon, to find the Time of that Star's Rising and Setting.*

O 2

The

The Time of culminating —  $\frac{1}{2}$  its Continuation, gives its Rising, and +  $\frac{1}{2}$  its Continuation, gives its Setting.

PROB. XV.

*TO find the Degree of the Ecliptick which rises or sets with a given Star, and to find the Time of that Star's Rising or Setting cosmically and achronically.*

Bring the Star to the East-side of the Horizon, seek the then rising Point of the Ecliptick on the wooden Horizon, and, in the Kalendar adjoined to it, you have the Time of the Star's Rising cosmically; the then setting or descending Point of the Ecliptick, gives the Time of the Star's Rising achronically, &c.

PROB. XVI.

*TO find the heliack Rising and Setting of any Star.*

Bring the Quadrant of Altitude to the West-side of the Meridian, the given Star to the East-side of the Horizon,

rizon, and the Twelfth Degree of Altitude on the Quadrant to the Ecliptick, the Point of the Ecliptick below that Degree, is elevated  $12^{\circ}$  above, and its opposite Point depressed  $12^{\circ}$  below the Horizon; that opposite Point on the wooden Horizon, shews in the Kalendar the Time of the heliac Rising of any Star of the first Magnitude.

The Converse of this Operation gives the Time of the heliac Setting.

If the Star be of the Second Magnitude, allow  $13^{\circ}$  of Depression, if of the Third,  $14^{\circ}$ . &c.





tion, and the Twelfth Degree of Altitude on the Quadrant to the Right, pick the Point of the Horizon below that Degree, is elevated  $12^{\circ}$  above, and its opposite Point depressed  $12^{\circ}$  below the Horizon; that opposite Point on the wooden Horizon, shows in the Almanac the Time of the Rising of any Star of the first Magnitude, and the Time of its Setting. The Converse of this Operation gives the Time of the Rising and Setting of the Star of the Second Magnitude, allowing  $12^{\circ}$  of Depression, in the Third,  $18^{\circ}$  in the Fourth, &c.





PLAIN DIALING  
BY THE  
GLOBES.





PLAIN DIALING

BY THE

G. L. ROBERTS.





## BY THE

**D**IALS take their Names from the Circles of the Sphere to which their Plans are parallel, or with whose Plans they coincide.

I. The *Hour-lines* are the common Sections of the Hour-circles of the Sphere with those Plans.

II. The *Stile* or *Gnomon* is a Line parallel to the common Section of the Plans of all the Hour-circles of the Sphere, *i. e.* parallel to the Axis of the World. Hence,

The Elevation of the Stile above the Dial-plan must be always equal to the Elevation of the Axis of the World, above the Plan of the Circle to which the Dial-plan is parallel; and the

the Stile must always be directed towards the elevated Pole.

III. An *Horizontal Dial* is, whose Plan is parallel to the Horizon.

IV. A *Vertical Dial* is, whose Plan coincides with that of some vertical Circle.

V. A *direct erect Vertical Dial* is, whose Plan is perpendicular to the Horizon, and whose Face looks directly South or North.

VI. An *erect declining Vertical Dial* is, whose Plan is perpendicular to the Horizon, but its Face declines from South or North, Eastward or Westward.

VII. A *direct reclining Dial* is, whose Plan looks directly South or North, but falls back from the Zenith.

VIII. A *direct reclining Vertical Dial*, whose Reclination is towards the same Parts with the Latitude of the Place, and equal to the Co-latitude, is called a *Polar Dial*, its Plan passing through the Poles of the World.

IX. A

IX. A *direct inclining Vertical Dial* is, whose Plan looks directly South or North, but makes an acute Angle with the Horizon.

X. If the Latitude of the Place, and the Inclination of the Plan be towards contrary Parts, and the Inclination equal to the Co-latitude, that Dial is called an *Equinoctial Dial*.

N. B. Every Dial-plate having Two Faces, it is plain the upper Face of the Equinoctial Dial shall have its Reclination equal to the Latitude, and the under Face of the Polar Dial an Inclination equal to the Latitude.

XI. From what has been said, it may be easily known what is meant by *reclining declining*, and *inclining declining Vertical Dials*.

XII. An *erect Vertical Dial*, whose Face looks directly East or West, is called a *Meridian Dial*.

XIII. The *Substilar* is the Line in the Dial-plan, upon which the Stile is set. It is the common Section of the Dial-plan, and the Plan of a great Circle passing through the Poles of the World, and those of the Plan.



Plate IV.  
FIG. 32.

## PROB. I.

*TO find the Inclination of a Plan.*

Let  $AB$  be a Plan inclined to the Horizon  $HR$ , apply to the Plan  $AB$  a Quadrant  $DCF$ , so as the Plummet  $CE$  may strain the Surface of the Quadrant, I say, the Arch  $DE$  is the Measure of the Angle of Inclination  $ABH$ . Draw  $BG \perp HR$ , because  $CE \parallel BG$ , the  $\angle^e ECF = CBG$ , but  $DCF = GBH$ , both being right Angles, the  $\angle^e DCF - ECF = GBH - CBG$ , *that is*,  $DCE = ABH$ . *Q. E. D.*

Plate IV.  
FIG. 33.

## PROB. II.

*TO find the Reclination of a Plan.*

Let  $AB$  be the reclining Plan. Draw  $BG \perp HR$ , representing the prime Vertical; so  $ABG$  is the Angle of Reclination. Raise  $KL \perp AB$ , apply a Quadrant  $CDF$  to  $KL$ , so as the Plumline  $CE$  may strain the Face of the Quadrant, I say,  $DE$  is the Measure of the Angle of Reclination  $ABG$ .

In

In the right angled Triangle  $NKB$ , the  $\angle^{les} BNK + NBK =$  a right Angle  $= DCF$ ; but, because  $CE \parallel BG$ , the  $\angle^{le} ECF = BNK$ , therefore  $DCE = NBK$ . Q. E. D.

PROB. III.

Plate IV.  
FIG. 34.

TO find the Declination of any Plan.

Take a Piece of Board, whose upper Surface is a right angled Parallelogram, as  $DB$ , on it describe a Circle  $c\alpha z$ ; on the Center  $c$  erect a Pin perpendicular. Draw  $FG \parallel BE$ ,  $MN \perp BE$ ,  $LN \perp FG$ , and place the Plan  $DB$  horizontally, with the Side  $BE$  applied to the Dial-plate. Observe when the Shadow of the Top of the Pin is on the Periphery of the Circle, as at  $z$  in the Forenoon, and at  $\alpha$  in the Afternoon the same Day; bisect the Arch  $\alpha z$  with the Diameter  $KL$ , 'tis plain  $KL$  is the Meridian, and  $KLN = KMN$ , is the Angle of Declination.

PROB.

## PROB. IV.

**T**O find the Distances of the Hour-lines on an horizontal Plan.

It is plain the brazen Meridian represents the Line of 12.

Rectify the Globe to the Latitude of the Place, bring the Equinoctial Colure to the brazen Meridian, turn the Globe Westward, until  $15^{\circ}$  of the Equator pass the Meridian; the Arch of the Horizon, intercepted betwixt the Meridian and Colure, is the Measure of the Angle formed by the Line of 12. and the Line of 1. at the Center of the Horizon. Again, turn the Globe Westward, until  $30^{\circ}$  of the Equator pass the Meridian, so you have the Angle of the Lines of 12 and 2. as before.

The same Hour-distances serve for the Forenoon.

The Substilar is the same with the Line of 12. and the Elevation of the Stile is equal to the Latitude of the Place.

PROB.



## PROB. V.

*TO find the Hour-distances on a direct erect Vertical Plan.*

Rectify the Globe to the Latitude of the Place, bring the Equinoctial Colure to the brazen Meridian, and the Quadrant of Altitude to the East Point of the Horizon, turn the Globe Eastward, until  $15^{\circ}$  of the Equator pass the Meridian; the Arch of the Quadrant, intercepted betwixt the Meridian and Colure, is the Measure of the Angle of the Line of 12. and Line of 11. at the Center of the vertical Circle, &c.

The same Hour-circles serve for Afternoon. The Substilar is the same with the Line of 12. and the Elevation of the Stile equal to the Co-latitude.

Direct reclining Vertical Dials are the same with direct erect Vertical Dials, in a Latitude equal to the Latitude of the Place  $\pm$  the Reclination, according as the Reclination is to-

wards

wards the same or contrary Parts with the Latitude.

The same holds in direct inclining Vertical Dials.

PROB. VI.

**T**O find the Hour-distances on an erect declining Vertical Plan, declining from South Westward, suppose  $25^{\circ}$ .

1. Rectify the Globe to the Latitude of the Place, bring the Quadrant of Altitude to  $25^{\circ}$  of the Horizon (reckoning from East toward South) and bring the Equinoctial Colure to the brazen Meridian, turn the Globe Eastward, until  $15^{\circ}$  of the Equator pass the Meridian; the Arch of the Quadrant, intercepted betwixt the Colure and Meridian, is the Measure of the Angle made by the Line of 12. and Line of 11. at the Center of the declining Plan, &c.

2. Bring the Quadrant to  $25^{\circ}$  of the Horizon from West Northward, and the Equinoctial Colure to the brazen Meridian, turn the Globe Westward,

ward, till  $15^{\circ}$  of the Equator pass the Meridian; so you have, as before, the Hour-distances from 12 to 1. &c.

3. Keep the Quadrant in the same Position, bring the Equinoctial Colure to  $25^{\circ}$  from South Westward, the Arch of the Quadrant, cut off by the Colure, gives the Angle made by the Substilar and Line of 12. and the Arch of the Colure, intercepted betwixt the Quadrant and Pole of the World, gives the Elevation of the Stile.

We shall refer declining reclining, and declining inclining Vertical Dials, until we come to the Projection of the Sphere, as being more easily done that Way.

PROB. VII.

Plate IV.  
FIG. 35.

*TO describe a Polar Dial.*

**I** Draw the horizontal Line  $mk$ , upon which raise the Perpendicular  $nd$  to any Length, upon  $nd$  as a Center, with  $nd$  as a Radius, describe a Semicircle  $ndl$ , which divide into Twelve equal Parts, through the



Center 12. and each Division, draw Lines 12 E, 12 F, &c. So you have the Hour-points E, F, G, K, &c. on the Horizontal Line MK. Lines drawn through these Points, parallel to 12 D are the Hour-lines.

A Pin, erected at D perpendicular to the Plan, and equal to 12 D, is the Gnomon, pointing the Hour with the the Shadow of its Top.

Or a Plate erected perpendicularly on 12 D as a Subtilar, having its Altitude equal to 12 D, and its upper Side  $\parallel$  12 D is the Stile.

Plate IV.  
FIG. 36.

### PROB. VIII.

*TO describe a Meridian Dial.*

Draw the horizontal Line MB, and make the Angle MBP equal to the Co-latitude of the Place, so BP is the common Section of the Equator and Meridian Plan, raise DC  $\perp$  BP, and in it take any Point c for a Center, and on it, with cD as a Radius, describe a Semicircle NDL, which divide into Twelve equal Parts, so you may have the Hour-points E, F, &c.  
in

fig. 2



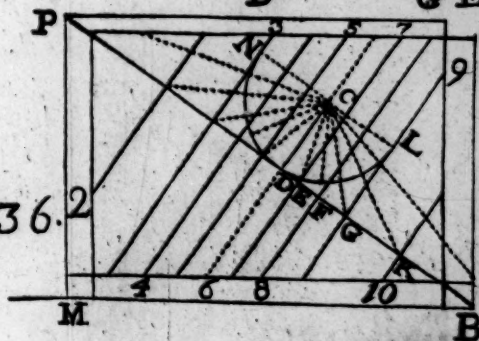
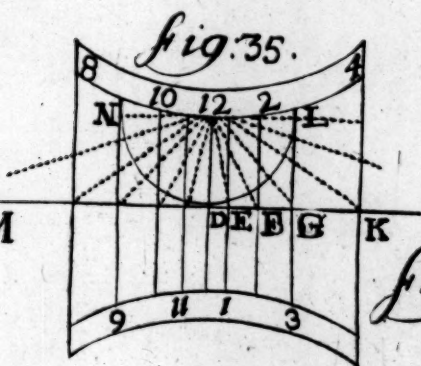
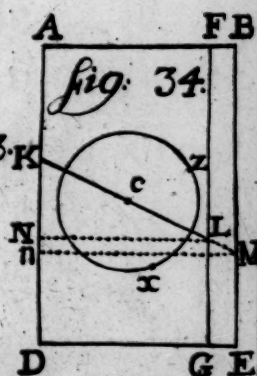
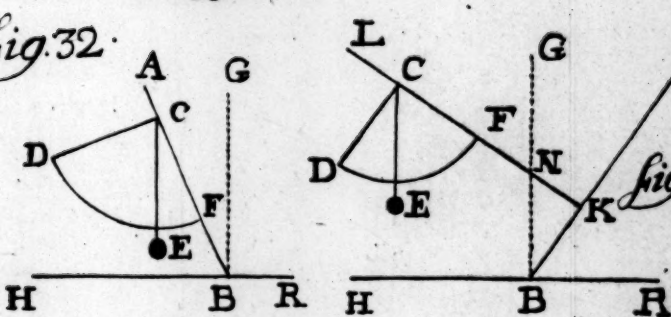
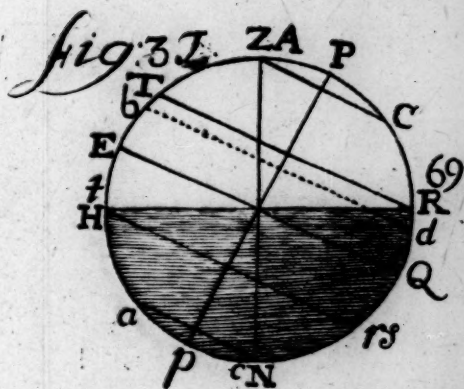
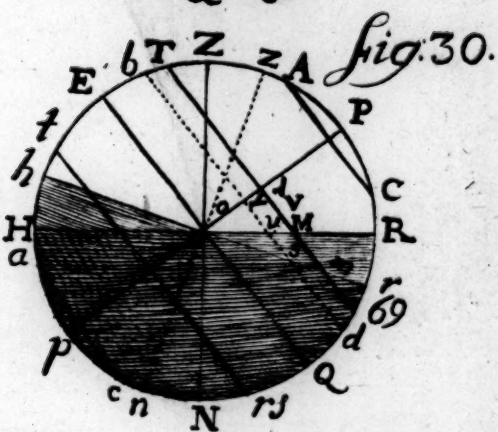
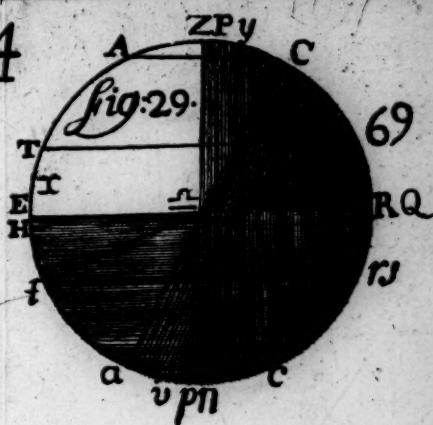
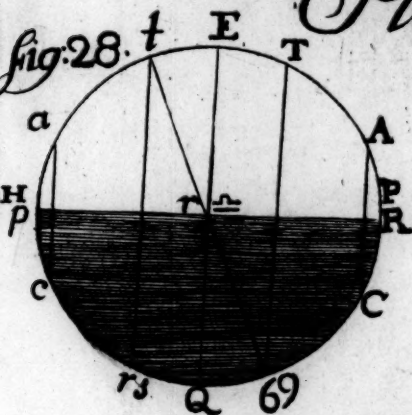
fig. 32



H



# Plate 4





in the Line  $BP$ . The Hour-lines are Lines drawn through these Points parallel to  $CD$ .

The Stile is a Pin equal to  $CD$ , erected perpendicular to the Plan at  $D$ , as in the Polar Dial.

The Problems of the Sphere may be solved by *spherique Trigonometry*: By it too we may calculate the Hour-distances on the several Dial-plans; which Calculation we shall refer to the Projection of the Sphere on the Plan of the Horizon.



by the Circle.  
 in the Line a. The Horizontal  
 Lines drawn through these Points  
 called a. c. d.  
 The Style is a Pin equal to c. d.  
 erected perpendicular to the Plan at  
 in the Polar Dial.

The Problems of the Sphere may  
 be solved by Spherical Trigonometry.  
 By it too we may calculate the Moon's  
 distance on the several Dial-Plans;  
 which Calculation we shall refer to  
 the Projection of the Sphere on the  
 Plan of the Horizon.





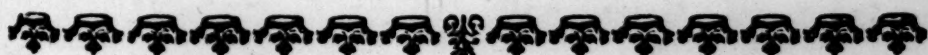
# RESOLUTION

O F

Most PROBLEMS of the *SPHERE*,

B Y

Spherique Trigonometry.







# RESOLUTION

OF  
MOST PROBLEMS OF THE SPHERE,

BY  
Spherical Trigonometry.





# RESOLUTION

O F

Most PROBLEMS of the *SPHERE*,

B Y

Spherique Trigonometry.

## LEMMA I. PROB. I.

*TO find the Sun's Place in the E-cliptick.*

Keep in Mind the Days on which the Sun enters the several Signs, which are these,

|                                         |                                        |                                        |
|-----------------------------------------|----------------------------------------|----------------------------------------|
| <sup>♈</sup><br>March 10.               | <sup>♉</sup><br>April 9.               | <sup>♊</sup><br>May 10.                |
| <sup>♋</sup><br>June 11.                | <sup>♌</sup><br>July 12.               | <sup>♍</sup><br>August 12.             |
| <sup>♎</sup><br>Sept <sup>r</sup> . 12. | <sup>♏</sup><br>Oct <sup>r</sup> . 12. | <sup>♐</sup><br>Nov <sup>r</sup> . 11. |
| <sup>♑</sup><br>Dec <sup>r</sup> . 11.  | <sup>♒</sup><br>January 9.             | <sup>♓</sup><br>February 8.            |

If, at the Day given, the Sun be in the Sign proper to the given Month, sub-

# 128 *Spherique Trigonometry.*

$$\begin{array}{r}
 \text{°} \quad \text{' } \quad \text{" } \\
 \quad \quad 59 \quad 8 \\
 \hline
 5 \quad 54 \quad 48 \\
 \quad \quad \quad 2 \\
 \hline
 11 \quad 49 \quad 36 \\
 + \quad 59 \quad 8 \\
 \hline
 \text{☾} \quad 12 \quad 48 \quad 44
 \end{array}$$

13. and  $13 \times 59' . 8'' = 12^{\circ} . 48' . 44''$ .

If the Sun be

$$\begin{array}{r}
 31 \\
 - 12 \\
 \hline
 19 \\
 + 6 \\
 \hline
 25 \text{ Days.} \\
 \hline
 \text{°} \quad \text{' } \quad \text{" } \\
 \quad \quad 59 \quad 8 \\
 \quad \quad \quad 5 \\
 \hline
 4 \quad 55 \quad 40 \\
 \quad \quad \quad 5 \\
 \hline
 \text{☾} \quad 24 \quad 38 \quad 20
 \end{array}$$

subtract the Day the Sun entred that Sign from the Day given; multiply the Remainder into  $59' . 8''$ . so you have the Degree of that Month's Sign the Sun is in.

So *August 25.* the Sun is in  $\text{☾} \quad 12^{\circ} . 48' . 44''$ . for  $25 - 12 = 13$ . and  $13 \times 59' . 8'' = 12^{\circ} . 48' . 44''$ .

If the Sun be in a Sign different from the proper Sign of the Month, from the Number of Days contained in the preceeding Month subtract the Day of the Sun's Entering into that Month's Sign, to the Remainder add the Day of the Month given; that Sum multiplied into  $59' . 8''$ . gives the Degree the Sun is in.

So



# *Spherique Trigonometry.* 129

So *August* 6. the Sun is in *Leo*  $24^{\circ}$ .  
 $38'.20''$ . for  $31 - 12 + 6 \times 59'.8'' =$   
 $24^{\circ}.38'.20''$ .

## LEMMA II. PROB. II.

*Given the Sun's Place, to find the nearest Equinoctial Point.*

Divide the *Ecliptick* into Four Quadrants, thus,

1. Q.  $\gamma$ .  $\delta$ .  $\Pi$ .
2. Q.  $\epsilon$ .  $\zeta$ .  $\text{III}$ .
3. Q.  $\simeq$ .  $\text{III}$ .  $\text{I}$ .
4. Q.  $\text{V}$ .  $\text{III}$ .  $\text{X}$ .

While the Sun is in the first and last Quadrants,  $\gamma$  is the nearest Equinoctial Point, and when in the middle Two  $\simeq$ .

In the First Quadrant, the Sun's Longitude is the Distance from the nearest Equinoctial Point.

In the Second Quadrant,  $180^{\circ}$ . — the Sun's Longitude is the Distance.

In the Third Quadrant, the Sun's Longitude —  $180^{\circ}$ . is the Distance.

In the Fourth Quadrant,  $360^{\circ}$ . — the Sun's Longitude is the Distance.

R

PROB.

Plate V.  
FIG. 37.

## PROB. III.

*TO find the Sun's Declination and right Ascension.*

1. Let  $p E p Q$  be the solstitial Colure,  $E Q$  the Equator,  $\odot \varpi$  the Ecliptick,  $\odot$  the Sun's Place,  $\gamma \odot$  or  $\sphericalangle \odot$  his Distance from the next Equinoctial Point,  $p D p$  a Meridian passing through  $\odot$  the Sun's Place, 'tis plain  $^s \gamma \odot . ^s \gamma \odot^a :: ^s \odot E . ^s \odot D$ . that is,  $R . ^s \odot^s$  Dist. ab Equinoct. ::  $^s$  greatest Decl. . to  $^s$  present Decl.

<sup>a</sup> 28. Trig.  
P. IV.

<sup>b</sup> 29.

2.  $^T E \odot . ^T D \odot^b :: ^s \gamma E . ^s \gamma D$ . that is,  $^T$  greatest Decl. .  $^T$  Decl. ::  $R . ^s$  Dist. ab Equinoct. on the Equator.

In the First Quadrant,  $\gamma D$  is the right Ascension.

In the Second Quadrant,  $180^\circ - \gamma D$  is the right Ascension.

In the Third Quadrant,  $180^\circ + \gamma D$  is the right Ascension.

In the Fourth Quadrant,  $360^\circ - \gamma D$  is the right Ascension.

PROB.

PROB. IV.

Plate V.  
FIG. 38.

*TO find the Sun's ortive or occidental Amplitude, and his ascensional Difference.*

Let  $ZHN$  be the Meridian of the Place,  $HR$  its Horizon,  $EQ$  the Equator,  $crl$  the Ecliptick,  $\odot$  the Sun's Place, draw the Meridian  $p\odot p$ , that cuts off the right Ascension at  $d$ , and  $f$ , the Point of the Equator that comes to the Horizon with the rising Sun, cuts off the oblique Ascension, so  $Fd$  is the ascensional Difference, and  $F\odot$  the Amplitude; 'tis plain  ${}^{\tau}RQ \cdot {}^{\tau}\odot d^a \cdot {}^{29. Trig.} P. IV. :: R \cdot {}^sFd. i. e. {}^{\tau}Co-lat. \cdot {}^{\tau}Decl. :: R \cdot {}^sascensional Diff. and {}^sRQ \cdot {}^s\odot d^b :: R \cdot {}^b 28. {}^sF\odot$ .

PROB. V.

Plate V.  
FIG. 39.

*Given the Sun's Altitude and Declination, and the Latitude of the Place, to find his Azimuth.*

In the Triangle  $p\odot z$  we have given  $p\odot$  the Co-declination,  $z\odot$  the Co-altitude, and  $pz$  the Co-latitude, sought the Angle  $pz\odot$  the Azimuth.

$R \cdot 2$

${}^{\tau}\frac{1}{2}p\odot$ .



# 132 *Spherique Trigonometry.*

$$\begin{aligned} & \frac{T \frac{1}{2} P \odot \cdot T \frac{1}{2} \odot Z + ZP}{T \frac{1}{2} \odot D - PD} :: T \frac{1}{2} \odot Z - ZP, \\ & \frac{\frac{1}{2} P \odot + \frac{1}{2} \odot D - PD}{s Z \odot \cdot R :: s \odot D \cdot s \odot Z D} = \frac{OD}{PD}. \\ & s Z \odot \cdot R :: s \odot D \cdot s \odot Z D. \\ & s Z P \cdot R :: s P D \cdot s P Z D. \\ & \odot Z D + P Z D = P Z \odot. \text{ the Azi-} \\ & \text{muth. } Q. E. F. \end{aligned}$$

Plate V.  
FIG. 39

## PROB. VI.

*Given the Sun's Altitude, Azimuth and Declination, to find the Hour of the Day in any given Latitude.*

<sup>a</sup> 40. Trig.  
P. IV.

In the  $\Delta^{lc} P Z \odot$ ,  $s P \odot \cdot s P Z \odot^a :: s Z \odot \cdot s Z P \odot$ , the Measure of the Hour from 12.

Plate V.  
FIG. 40.

## PROB. VII.

*TO find the Beginning of the Morning, and Ending of the Evening Twilight.*

The Depression  $G O$ , and Declination  $d \odot$  being known,  $N \odot$  and  $p \odot$  are easily found, and  $p N$  is equal to the Co-latitude of the Place, sought the Angle  $\odot p N$ , the Measure of the Hour from Midnight.

$$T \frac{1}{2} N \odot.$$

# Spherique Trigonometry. 133

$$T_{\frac{1}{2}}N \odot . T_{\frac{1}{2}}P \odot + PN :: T_{\frac{1}{2}}P \odot - PN.$$

$T_{\frac{1}{2}}N.$

$$T_{\frac{1}{2}}N \odot + \frac{1}{2}N = \frac{OD}{PD}.$$

$$\odot P . R :: \odot D . \odot P D.$$

$$\odot P N . R :: \odot N D . \odot N P D.$$

$\odot P D + N P D = \odot P N$ , the Hour  
from Midnight. Q. E. F.

## LEMMA III. PROB. VIII.

Plate V.  
FIG. 41.

*TO find the Declination of any Star  
by Observation.*

Let HR be the Horizon, EQ the  
Equator, P the Pole of the World,  
Z the Zenith, s,  $\sigma$ ,  $\Sigma$ , or  $\int$  the Star,  
observe the Star's Meridian Altitude,  
HS, H $\sigma$ , R $\int$ , R $\Sigma$ ; 'tis plain HS — HE  
= ES the Declination North, HE —  
H $\sigma$  = E $\sigma$ , R $\Sigma$  — RP = P $\Sigma$  the Co-  
declination, and RS + RQ = QS the  
Declination.

## LEMMA IV. PROB. IX.

*Given the Sun's right Ascension, to  
find the right Ascension of any fixt  
Star, by Help of a Pendulum Watch.*

When

# 134 *Spherique Trigonometry.*

When the Sun is on the Meridian, set your Watch at 12. observe the Hour and Minute when the Star comes to the Meridian, that Time reduced to Degrees of the Equator, is the Difference of their right Ascensions.

Plate V.  
FIG. 42.

## PROB. X.

*Given the Declination and right Ascension of any Star, to find its Longitude and Latitude.*

Make EQ the Equator, CL the Ecliptick, P, Z their Poles, and S the Star; its Declination AS being given, PS is known, and  $90^\circ - \gamma A$ , the Star's Distance from the nearest Equinoctial Point, measured on the Equator,  $= EA = \angle^{\text{le}} ZPS$ .

In the  $\Delta^{\text{le}} ZPS$ ,  ${}^{\text{t}}ZP.R :: {}^{\sigma}ZPD.{}^{\text{t}}PD$ .  
 $SP - PD = SD$ .

<sup>a</sup> 32. Trig.  
P. IV.

$\sigma PD. \sigma SD^a :: \sigma PZ. \sigma SZ$ . Co-lat.

Likewise  ${}^sZS. {}^sZPS :: {}^sPS. {}^sPZS$ . which known, the Longitude is easily found. Q. E. F.

PROB.



PROB. XI.

Plate V.  
FIG. 43.

*TO find the ascensional Difference of any fixt Star, and consequently its oblique Ascension and Continuation above the Horizon.*

It is plain  $oA$  is the ascensional Difference, and  $s$  being the Star on the Horizon  $HR$ , and  $PSA$  being the Secondary to the Equator  $EQ$ ,  $TRQ$ .  
 ${}^TSA :: R. {}^sOA. i. e. {}^TCo-lat. . {}^TDecl. :: R. {}^sascensional\ Difference.$

That ascensional Difference subtracted from the right Ascension of the Star, if it declines towards the elevated Pole, or added to it, if otherwise, gives the oblique Ascension.

The same ascensional Difference added to, or subtracted from  $90^\circ$ . according as the Declination is towards the elevated or deprest Pole, gives the Semidiurnal Arch of that Star.

In the same Triangle the ortive Amplitude  $os$  is easily found.

PROB.

Plate V.  
FIG. 44.

PROB. XII.

**G**iven the oblique Ascension of any Star, to find its cosmick and achronick Rising.

Let  $s$  be the rising Star.

In the  $\Delta^{le} \gamma o A$ , we have given  $\gamma o$  from the oblique Ascension given, the  $\angle^{le} o \gamma A = 23^{\circ}. 30'$ . and  $EOA$  the Angle of the Equator with the Horizon.

Draw the Perpendicular  $\gamma G$ , and

$$^s \gamma \odot G . R :: \sigma \gamma \odot . ^s \odot \gamma G .$$

$$\odot \gamma A \frac{+}{2} \odot \gamma G = G \gamma A .$$

$$\sigma G \gamma A . \sigma G \gamma \odot ^2 :: \tau \gamma \odot . \tau \gamma A .$$

<sup>a</sup> 34. Trig.  
P. IV.

So  $A$ , the Point of the Ecliptick that rises with the Star, is known; when the Sun then comes to that Point, the Star rises cosmically, and when he comes to the opposite Point, he rises achronically.

In the same Triangle, the  $\angle^{le} \gamma A o$  is easily found.

PROB.

PROB. XIII.

Plate V.  
Fig. 44.

*TO find the heliack Rising and Setting of any Star.*

Let  $D$  be the Sun's Place while the Star  $s$  rises heliacally.

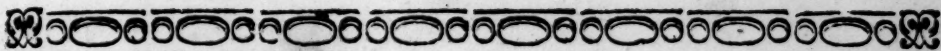
In the  $\Delta^{lc} ADF$ , right angled at  $F$ , we have given  $FD$  the Depreffion, and the  $\angle^{lc} FAD$ , that the Ecliptick makes with the Horizon, and  $^sFAD. ^sFD :: R. ^sAD$ . so  $\gamma A + AD = \gamma D$ , gives the Sun's Place when the Star rises or sets heliacally.



S



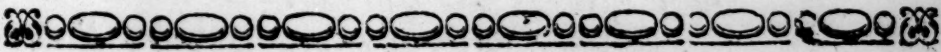




# Spherique Geometry,

CONTAINING

The STEREOGRAPHICK and  
ORTHOGRAPHICK PRO-  
JECTIONS of the  
*SPHERE.*

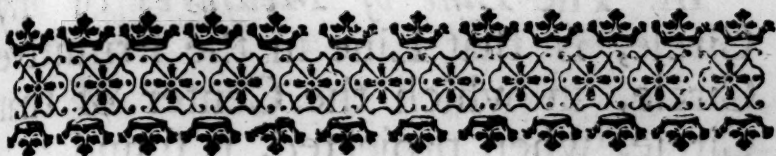


Spherical Geometry

CONTAINING

The Stereographic and  
Orthographic Projections  
of the Sphere





# Spherique Geometry,

CONTAINING

The STEREOGRAPHICK and OR-  
THOGRAPHICK PROJEC-  
TIONS of the *SPHERE*.

---

## DEFINITIONS.

I. **S**TEREOGRAPHICK *Projecti-*  
*on* represents the several Cir-  
cles of the Sphere, on a Plan  
passing through its Center, in such  
Sort as they would appear to the Eye  
were it placed on the Surface of the  
Sphere, and at the Pole of that Cir-  
cle on whose Plan the other Circles  
are projected.

II. That Circle, on whose Plan the  
other Circles are projected, is called  
the *Primitive Circle*,

III. The

## 142 *Spherique Geometry.*

III. The *Line of Measures* of any Circle, is that Line in the Primitive Circle, in which the Center of that Circle, when projected, is found.

Plate V.  
FIG. 47, 48.

### PROP. I.

*Every Circle on the Sphere, whether parallel or oblique to the Primitive, is a Circle in the Projection.*

Make  $BC$  the Circle to be projected on the Plan of the Primitive  $EG$ , at whose Pole  $A$  suppose the Eye to be placed, then shall  $B$  be projected into  $b$ , and  $c$  into  $c$ ; and because  $AD = AC$ , the  $\angle^e ACD = ABC$ , therefore the  $\Delta^e ABC$  is subcontrarily cut by  $DC$  or  $bc$ , therefore  $bc$  is a Circle. Q. E. D.

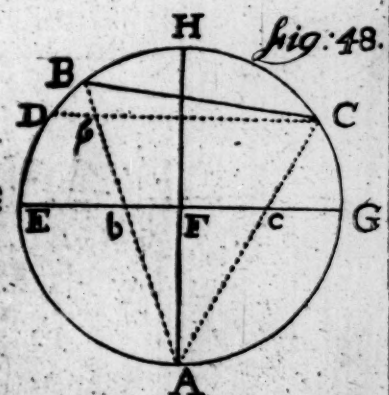
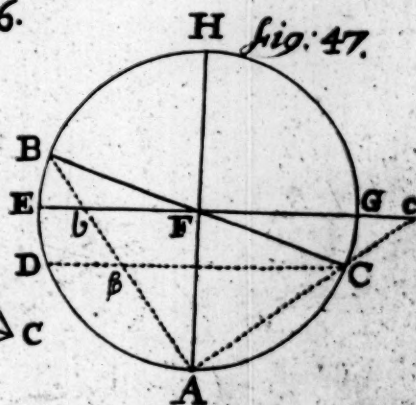
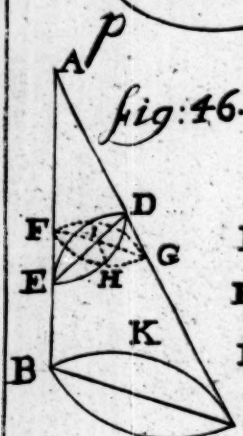
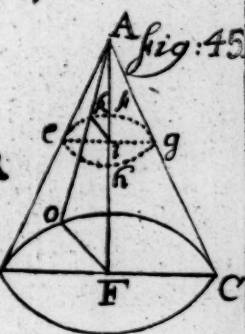
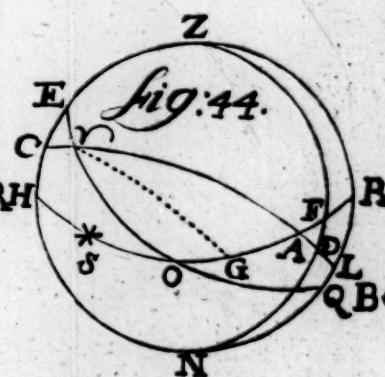
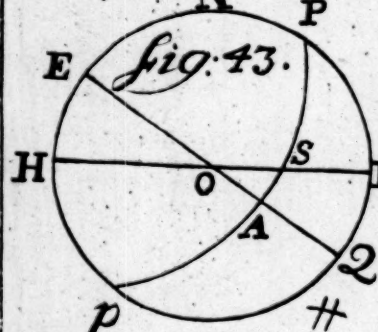
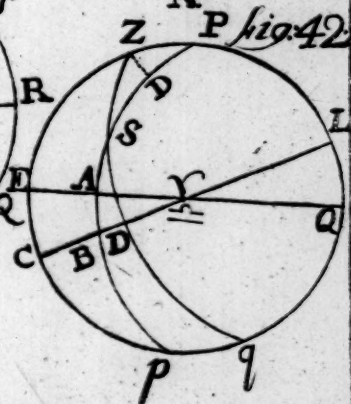
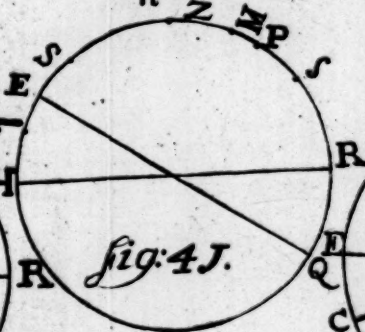
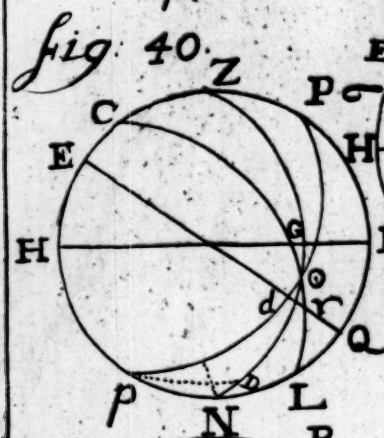
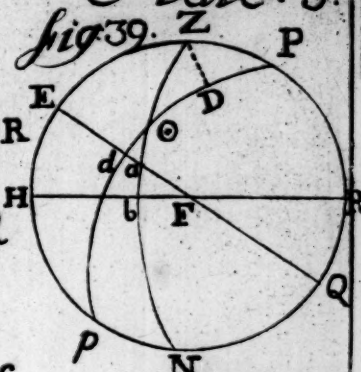
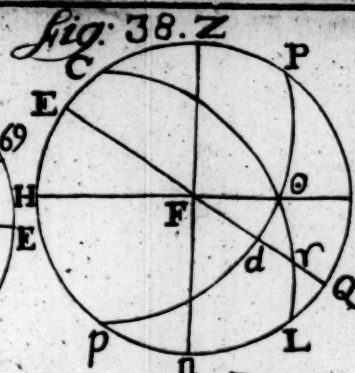
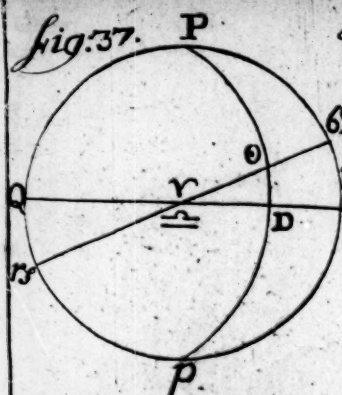
Plate VI.  
FIG. 49.

### PROP. II.

*The Distance of the Center of any great Circle from the Center of the Primitive, is equal to the Tangent of its Elevation above the Plan of the Primitive.*

Suppose

Plate. 5.





## Spherique Geometry. 143

Suppose the Eye at  $A$ , projecting the Circle  $BC$  on the Plan of the Primitive  $EG$ , the Point  $B$  appears in the Line of Measures at  $b$ , and  $Fb = \tau \frac{1}{2} BFH$ , the Point  $c$  appears at  $c$ , and  $Fc^a = \tau \frac{1}{2} HFc$ , and  $BAC$  is <sup>b</sup> a right <sup>Def. 10. Trig. P. I. 20. c. 3.</sup> Angle. Bisect  $bc$  the projected Diameter at  $i$ , that  $i$  is the Center of the projected Circle  $A b H$ , and  $Fi = FBF$  the Elevation of  $BC$  above the Primitive.

For, because  $AF^c \perp bc$ , the  $\Delta^{les} \text{ } AF^c, FAC, bAC^d$  are similar, and <sup>Def. 6. Trig. P. IV. 8. c. 6.</sup>  $BFH^e = bAF^d = AC^e = \frac{1}{2} AiF$ , <sup>20. c. 3.</sup> therefore  $BFH = AiF$ , therefore their Complements are equal, *viz.*  $BFE = FAi$ , whole Tangent is  $Fi$ . **Q. E. D.**

*Cor. 1.* The Radius of the projected Circle is equal to the Secant of its Elevation above the Plan of the Primitive  $bi = Ai$ .

2.  $bF$  the Distance of the projected Circle from the Center of the Primitive on the Line of Measures,  $= bAF = \tau \frac{1}{2} BFH$  the Co-elevation.

3. Set off  $Hm = BF$ , draw the Diameter  $lm$ , that shall be the Axis of  
 $BC$ ,



# 144 *Spherique Geometry.*

B C, and  $l, m$ , its Poles, joyn  $A m$ ; 'tis plain the Pole  $m$  shall appear in the Line of Measures at  $p$ , and  $F p$  its Distance from the Center of the Primitive,  $= \tau_{F A P} = \tau_{\frac{1}{2}}$  Elevation.

4. Hence, the Pole of any great Circle falls always betwixt the Center of that Circle, and that of the Primitive.

*Praxis.* In projecting oblique great Circles, reckoning the Distances from the Center to the Primitive, then the Distance of the Center  $= \tau$  Elevation, the Distance of the Pole  $= \tau_{\frac{1}{2}}$  Elevation, the Distance of the projected Circle on the Line of Measures  $= \tau_{\frac{1}{2}}$  Co-elevation, and the Radius  $=$  Secant of the Elevation.

Plate VI.  
FIG. 50.

## PROP. III.

*Every great Circle perpendicular to the Primitive, is projected into a right Line, and is measured on the Line of Semitangents.*

Suppose the Eye at  $A$ , projecting the Circle  $E H G$  upon the Plan of the Primitive  $E G$ , the Points  $H, o, p, k, m$ , appear

# Spherique Geometry. 145

appear to the Eye at F, *a*, *b*, *d*, *e*, therefore the whole Arch *EHG* shall be projected into the Line *EG*. But  $fa = {}^TFAa = {}^T\frac{1}{2}HFO = {}^T\frac{1}{2}HO$ . Q. E. D.

## PROP. IV.

Plate VI.

FIG. 51.

*THE Distance of the Center of a lesser Circle, whose Pole is in the Plan of the Projection, from the Center of the Primitive, is equal to the Secant of the Distance of that lesser Circle from its Pole, and its Radius is equal to the Tangent of that Distance.*

Let the lesser Circle *bc* be to be projected by the Eye at *A*, upon the Plan *EG*, in which Plan the Pole *G* of the lesser Circle is found. It is plain *bc* shall be the projected Diameter, which bisected gives *i* for the Center,  $ib = i_B$  for the Radius.

The  $\Delta^{les} ABH$ ,  $CFH$ , having the  $\angle^{les}$  at  $^aB$  and  $^bF$  right, and the  $\angle^{le} H$  common, are similar, then we have  $\angle^{le} \overset{a}{3r. c. 3.} \overset{b}{Def. 6.} \overset{Trig. P. IV.}{\text{}} = \overset{c. 5. c. 1.}{\text{}} \angle^{le} FBb^c = FAB = FCB^c = i_{BC}$ ; but  $b_B i + i_{BC}^a = \underset{T}{\text{a right}} \angle^{le} = \underset{FBb}{b_B i +}$

# 146 *Spherique Geometry.*

$FBb = FBI$ , therefore  $Bi$  = the Radius  $bi$ , is the Tangent, and  $Fi$  the Secant of  $BG$ . *Q. E. D.*

Plate VI.  
FIG. 52, 54.

## PROP. V.

*TO describe a lesser Circle, whose Poles are not in the Plan of the Projection.*

Let the Circle  $BC$  be to be projected by the Eye at  $A$ , on the Plan  $EG$ ;  $bc$  shall be its projected Diameter, which bisected gives  $d$  for the Center. It is plain  $Fb$ ,  $Fc$  are the Semi-tangents of the Arches  $BH$ ,  $CH$ , *i. e.* of the Distances of the Extremities of its Diameter from the Pole of the Primitive.

Plate VI.  
FIG. 55.

## PROP. VI.

*IF Two Circles  $CBF$ ,  $ABI$ , cut each other at  $B$ , the  $\angle^{le} FBI$  made by the Circles =  $\angle^{le} ABC$  made by their Radii, at the Point of Intersection  $B$ .*

Draw the Tangents  $BD$ ,  $BE$ . Because the infinitely small Parts of the Circles at  $B$ , coincide with  $BD$ ,  $BE$ , there-



therefore the curved Angle  $FBI = DBE =$  the right  $\angle^{le} DBC - EBC =$  the right  $\angle^{le} EBA - EBC = CBA$ . *Q. E. D.*

PROP. VII.

Plate VI.  
FIG. 53.

*ALL great Circles passing through the Point b have their Centers in the Line IL, EI the Line of Measures to the Circle ABH perpendicular to the Circle EG, and passing through I its Center.*

The Center of the Circle  $QbR$  is in the Perpendicular  $IL$ , as at  $K$ .

For all great Circles on the Sphere <sup>a I. Trig. P. IV.</sup> cutting each other at Points diametrically opposite, their Representatives in the Projection must cut at the same projected Points, suppose  $b, c$ , now the  $\Delta^{les} kib, kic$  have the Angles at <sup>b 15. e. 1.</sup>  $i$  right,  $ib^b = ic$ , and  $ik$  common, therefore  $kb, kc$  being equal, are the Radii, and  $k$  the Center of the Circle  $QbR$ . *Q. E. D.*

*Schol.* The Distance  $ik$  of the Center  $k$ , from  $i$  the Center of the Perpendicular  $AbH$ , is equal to the Tan-  
T 2 gent

# 148 *Spherique Geometry.*

c 6 b.

gent of the Angle  $\text{H}b\text{R}$ , answering to the Radius  $ib$ , for  $ik =$  the Tangent of the  $\angle^{\text{le}} ib\text{K}^{\text{c}} = \angle^{\text{le}} \text{H}b\text{R}$ . Q. E. D.

Plate VI.  
FIG. 55.

## PROP. VIII.

*THE Angles made by Circles on the Sphere, are equal to the Angles made by their Representatives on the Plan of the Projection.*

<sup>a</sup> 32. e. 3.  
<sup>b</sup> 5. e. 1.  
<sup>c</sup> 29. e. 1.  
<sup>d</sup> 6. c. 1.

<sup>e</sup> 4. e. 1.

Suppose the Eye at  $\text{A}$ , projecting the  $\angle^{\text{le}} \text{RBS}$  on the Plan  $\text{EF}$ ; draw  $\text{BD}$ ,  $\text{BC}$  Tangents to the Two Circles at  $\text{B}$ . The Plan  $\text{DBC}$  made by the Tangents, and  $\text{EF}$ , are both perpendicular to the Plan of the Circle  $\text{BQA}$ , therefore their common Section  $\text{DC}$  is perpendicular to that Plan. The Eye at  $\text{A}$  projects  $\text{DB}$  into  $\text{DF}$ , and  $\text{CB}$  into  $\text{CF}$ . I say, the  $\angle^{\text{le}} \text{DFC} = \angle^{\text{le}} \text{DBC}$ . Draw  $\text{BQ} \parallel \text{EF}$ , joyn  $\text{AQ}$ . The  $\angle^{\text{le}} \text{DBA}^{\text{a}} = \text{AQB}^{\text{b}} = \text{ABQ}^{\text{c}} = \text{BFD}$ , therefore  $\text{DB}^{\text{d}} = \text{DF}$ . In the  $\Delta^{\text{les}} \text{CDB}$ ,  $\text{CDF}$ , the Angles at  $\text{D}$  are right,  $\text{DB} = \text{DF}$ , and  $\text{CD}$  are common, therefore the  $\angle^{\text{le}} \text{DFC}^{\text{e}} = \angle^{\text{le}} \text{DBC}$ . Q. E. D.

PROP.

PROP. IX.

Plate VI.  
FIG. 57.

*IF a great Circle  $zPNP$  pass through the Poles of the Two great Circles  $\mathcal{A}Q$ ,  $HO$ , and if  $P$ ,  $N$ , the remotest Poles of these Circles, be joyned by a right Line  $PN$ , about which  $PN$ , as an Axis, suppose a Plan  $PDKN$  to move, the Arches  $KQ$ ,  $DO$  of the Circles  $\mathcal{A}Q$ ,  $HO$ , intercepted betwixt  $PDKN$ , and the Circle  $zPNP$  shall be always equal, as also the Arches of any Two Parallels  $\mathcal{A}q$ ,  $ho$  to these Circles.*

For  $PO = NQ$ ,  $PD = NK$ , and  $\angle^{le} OPD = QNK$  being the Inclination of the Two Plans, therefore  $DO = KQ$  the same Way  $do = kq$ . Q. E. D.

PROP. X.

Plate VII.  
FIG. 58.

*TO make the Scale of natural Sines, Tangents, Secants, Semitangents and Chords.*

Divide any Circle  $ADBE$  into Four Quadrants, and each Quadrant into  $90^\circ$ . tho' here we have only every  $10^\circ$  marked.

I. 'Tis



## 150 *Spherique Geometry.*

1. 'Tis plain, that Lines drawn from each Degree of the Quadrant  $ADLAC$  the Radius, shall mark the Sines of the several Arches on that Radius  $AC$ , thus  $n\alpha$  cuts off  $c\alpha^a = nF = 40^\circ$ .  
 \* 34. e. i. So  $CA$  is the Line of Sines.

2. Draw  $EH$  touching the Circle at  $E$ , Lines drawn through the Center  $c$ , and every Degree of the Quadrant  $AE$ , shall mark the Tangents of the several Arches on the Line  $EH$ , and shall themselves be Secants of these Arches, which Secants, being transposed to  $CA$  produced, gives  $CK$  for the Line of Secants, and  $EH$  is the Line of Tangents.

3. Through  $D$ , and each Degree of the Quadrant  $EB$ , draw right Lines, these shall cut off the Tangents of the Half of these Arches on the Line  $CB$ . So  $CB$  is the Line of Semitangents.

4. Draw  $DB$ , on  $D$  as a common Center with the Chords of the several Arches of the Quadrant  $DB$ , as Radii describe Arches, these shall mark the Chords of the several Arches of the Quadrant  $DB$ , on the Line  $DB$ . So  $DB$  is the Line of Chords.

PROP.



# Plate 6

fig: 49.

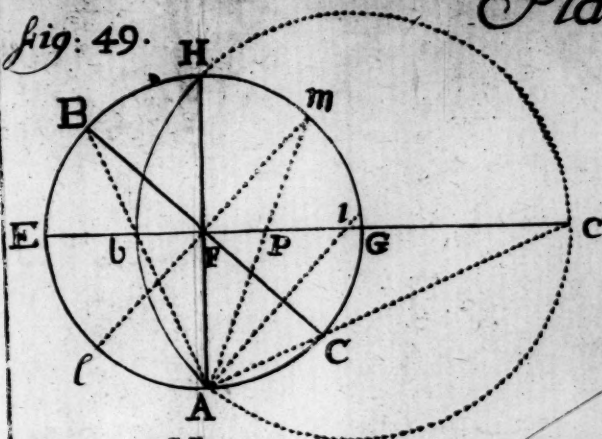


fig: 50.

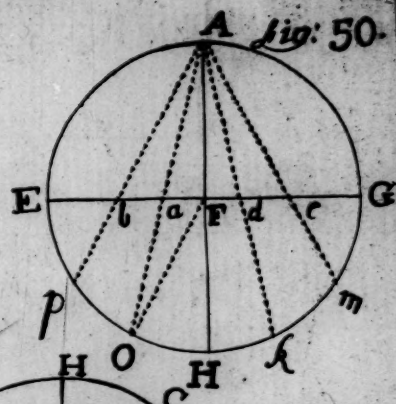


fig: 51.

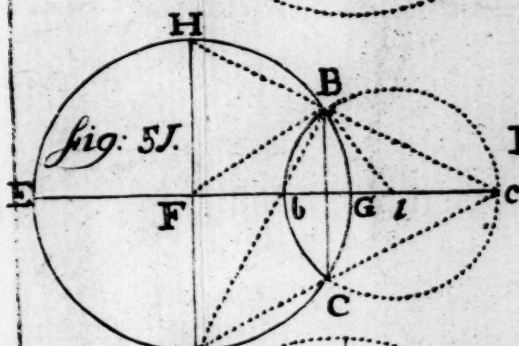


fig: 52.

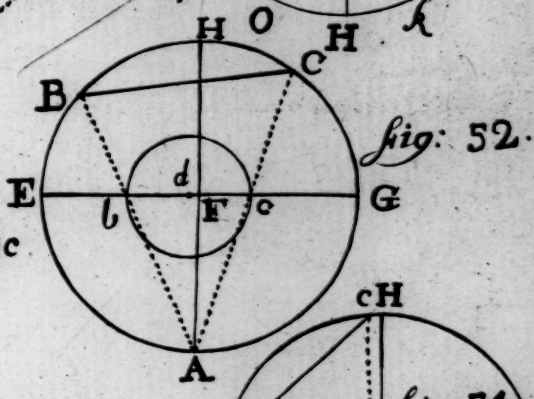


fig: 54.

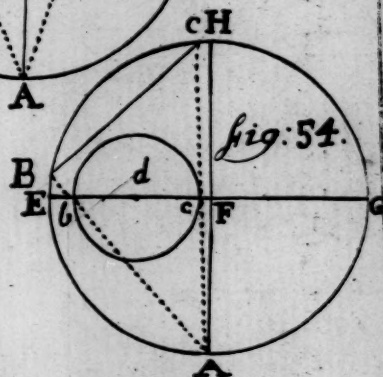


fig: 53.

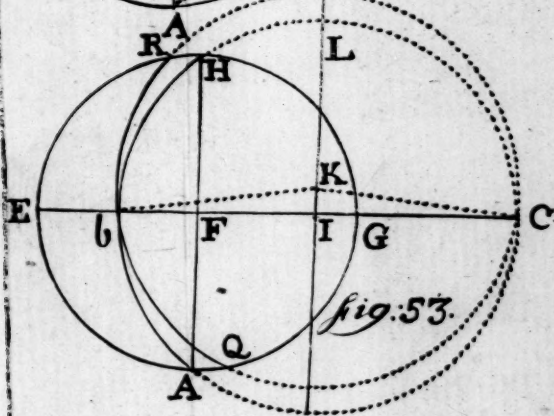


fig: 55.

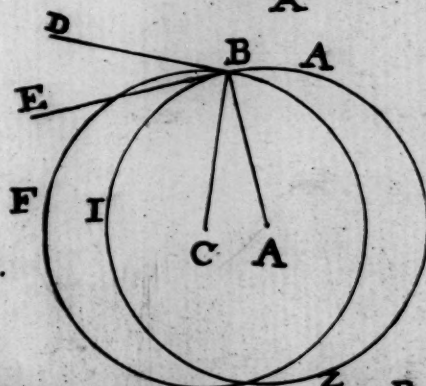


fig: 56.

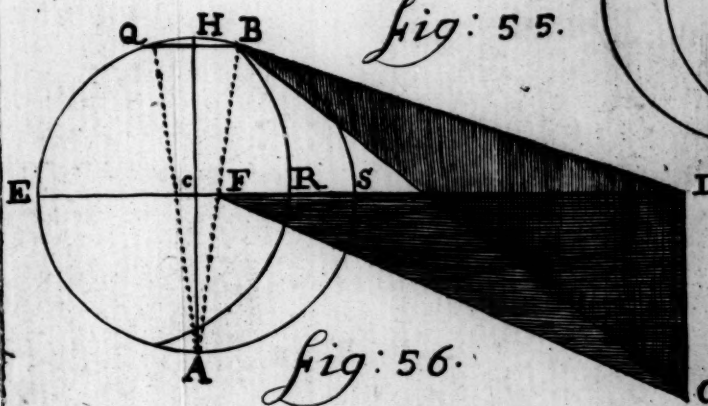
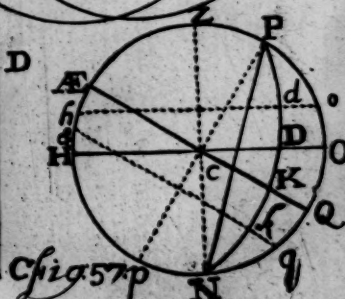


fig: 57.





PROP. XI. PROB. I.

*TO find the Poles of any Circle.*

1. The Poles and Center of the Primitive coincide.

2. The Extremities of a Diameter perpendicular to a right Circle, are its Poles.

3.  $r \frac{1}{2}$  Elevation of any oblique Circle, set off from the Center of the Primitive on the Line of Measures, gives the Pole of that oblique Circle.

PROP. XII. PROB. II.

Plate VII.  
FIG. 49.

*TO lay down on the Projection any Angle required.*

1. If the Center of the Primitive be the angular Point, the Case is the same as in plain Angles.

2. If the angular Point be in the Periphery of the Primitive, as at A, through A, and the Center E, draw AC; on A and C, as Centers with the Secant of the Angle sought, strike Two Arches cutting one another at x,

that

# 152 *Spherique Geometry.*

that is the Center of the Circle, and  $HAC$  the Angle sought.

3. Let  $G$  be the angular Point, and the Angle to be made by oblique Circles, that Angle, as  $HGN$ , shall be made by *Schol.* PROP. VII.

4. At a Point  $G$  in a right Circle  $DEGF$ , to make, with that Circle, an Angle of any Number of Degrees, as  $68^\circ$ .

Draw  $HEK \perp DF$ , through  $K$  and  $G$  draw  $KM$ , set off  $Ei = {}^r FM$ , and  $IL \perp DF = {}^r 68^\circ$ . answering to the Radius  $GI$ , upon  $L$ , as a Center with the Radius  $LG$ , describe a Circle; it is done. For  $\angle^{le} HGD = 90^\circ$ . and  $NGH = 22^\circ$ . then  $DGN = 68^\circ$ . Q. E. F.

Plate VII.  
FIG. 6Q.

## PROP. XIII. PROB. III.

*TO draw a great Circle passing through a given Point  $P$ , and making any Angle, as  $HQP = 50^\circ$ . with the Primitive.*

Upon the Center  $E$ , with the Tangent of the given Angle, strike an Arch, and upon the given Point  $P$ , with the Secant of the same Angle, strike

# Spherique Geometry. 153

strike another Arch; their Point of Intersection  $z^a$  is the Center, and  $zP^a$  <sup>2 b</sup> the Radius of the Circle sought. Q. E. F.

## PROP. XIV. PROB. IV.

Plate VII,  
FIG. 61.

**TO** draw a great Circle passing through any Two Points F, G, within the Primitive.

Through either of the Points, as F, draw a Diameter AFEC, cross it at right Angles with the Diameter BD, joyn FB, and draw BILFB, and cutting AC produced at I, that Point I is a Third Point, through which the Circle must pass.

We have here only to prove that the Line HK, joyning the Points of Intersection with the Primitive, is a Diameter. If you say it is not, let HEM be a Diameter, meeting HGF continued at L, then  $EH \times EL^a = EF \times EI^b$  <sup>a 35. e. 3.</sup>  
 $= EB \times EL = EH \times EM$ , <sup>b Cor. 8. e. 6.</sup> therefore  $EM = EL$ . Q. E. A.

U

PROP.



Plate VII.  
FIG. 62.

PROP. XV. PROB. V.

*TO draw a great Circle perpendicular to a given great Circle.*

Draw it through its Poles.

1. If the oblique Circle  $AKC$  must be perpendicular to  $BD$  a right Circle, and pass through a given Point  $K$ , through  $K$ , and the Poles of  $BD$ ,  $A$ ,  $C$ , describe a Circle.

<sup>a</sup> 2 b.

<sup>b</sup> 1. Cor. 2.

2. If the Perpendicular  $AKC$  must make a given Angle with the Primitive, as of  $26^\circ$ . on the right Circle  $BD$ , from the Center  $E$ , set off  $Ex^a =$  Tangent of the given Angle, or from the Pole  $A$  to  $x$ , set off  $Ax^b =$  Secant of that Angle, so  $x$  is the Center.

<sup>c</sup> 14 b.

3. If to the great Circle  $AKC$  a Perpendicular must be drawn, so as to pass through a Point  $m$  given in the Circle  $AKC$ , through  $P$ , the Pole of  $AKC$ , and  $m^c$ , describe a Circle, and it is done.

4. If the Perpendicular  $FPG$  must make a given Angle, as of  $62^\circ$ . with

the

*Spherique Geometry.* 155

the Primitive, it is done by PROP. XIII.

PROP. XVI. PROB. VI.

*TO measure any Arch of a great Circle.*

1. The Primitive is measured on the Line of Chords.

2. Right Circles are measured on the Line of Semitangents.

3. A Ruler laid through the Pole of any oblique Circle, and the Divisions of the Primitive, cuts off the like Divisions on the oblique Circle, by PROP. IX.

PROP. XVII. PROB. VII.

Plate VII.  
FIG. 63.

*TO measure any spherique Angle.*

1. If the angular Point be in the Periphery of the Primitive, the Distance of the Center of the oblique Circle from that of the Primitive, is the <sup>a</sup> Tangent of the Angle.

2. If the Angle is within the Primitive, the Arch of the Primitive, intercepted betwixt Two Lines drawn  
U 2 from

# 156 *Spherique Geometry.*

from the angular Point through the Poles of the Circles forming the Angle, is the Measure of that Angle, so  $v$  and  $y$  being the Poles of the Circles  $ALC$ ,  $OLQ$ , the Arch  $de$  is the Measure of the  $\angle^{\text{le}} QLC$ .

Plate VII.  
FIG. 64.

## PROP. XVIII. PROB. VIII.

*TO measure any Arch of a lesser Circle.*

On the Center of the Primitive, with a Radius equal to the Semitan-  
gent of the Distance of the lesser Cir-  
cle from its furthest Pole, describe  
a Circle; through the Divisions of that  
Circle, and the nearest Pole of the  
lesser Circle, lay a Ruler, that cuts off  
the like Divisions of the lesser Circle,  
by PROP. IX.

*To project the Sphere on the Plan of  
the solstitial Colure.*

Plate VIII.  
FIG. 65.

**W**ith the Chord of  $60^\circ$ . describe  
the Primitive, draw the Dia-  
meter  $pp$ , representing the Equino-  
ctial Colure and Axis of the World.  
Draw





# Plate. 7.

fig. 58.

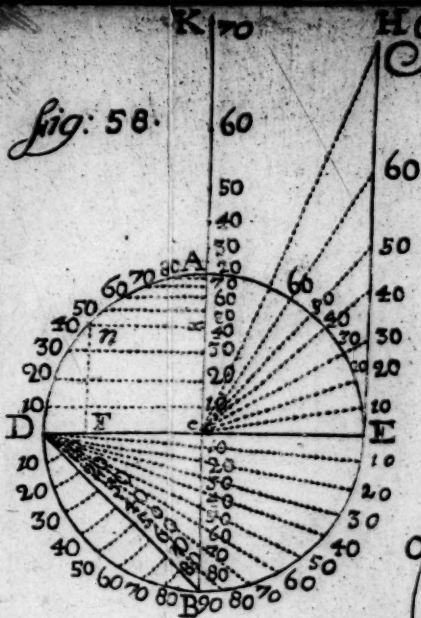


fig. 59.

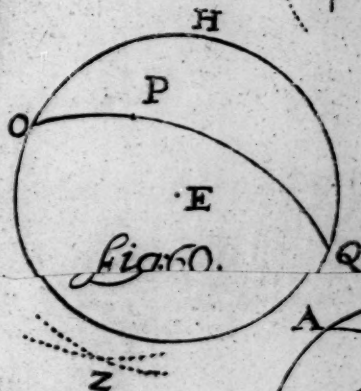
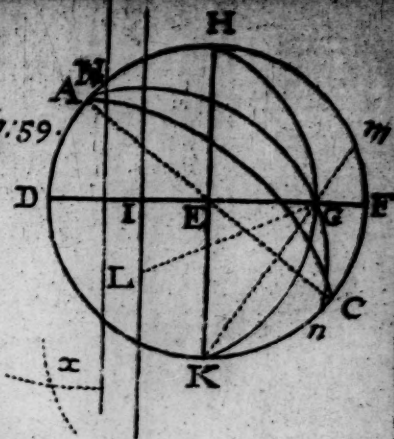


fig. 63.

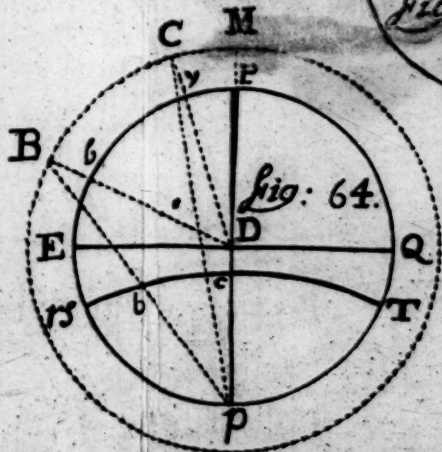
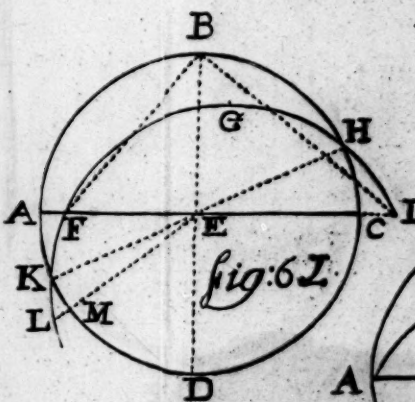
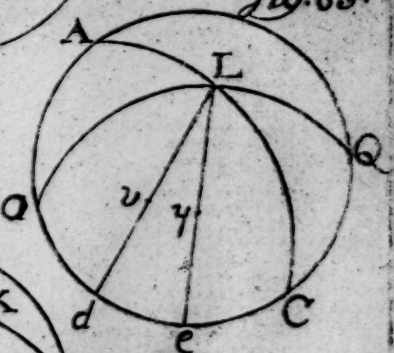
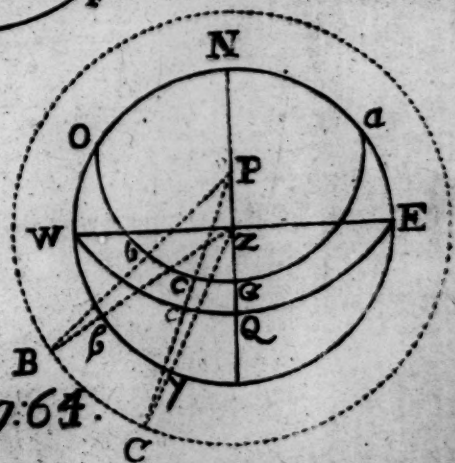


fig. 64.



# Spherique Geometry. 157

Draw the Equator  $EQ \perp PP$ , from  $E$  set off  $Ew = 23^{\circ}. 30'$ . joyn  $w$   $\odot$  for the Ecliptick. On  $PP$ , produced as a Line of Measures with a Radius  $b = 66^{\circ}. 30'$ . describe the Tropicks  $\tau \odot$ ,  $t w$  on the same Line of Measures with a Radius  $a = 23^{\circ}. 30'$ . describe the Polars  $Aa$ ,  $Bb$ , set off  $EH = 34^{\circ} = \text{Co-lat. Edinb.}$  joyn  $HR^a$  for the Horizon of *Edinb.*

Find the Sun's Place in the Ecliptick, as  $\odot$ , for any Time, draw  $^c$  the Meridian  $Pop$  cutting the Equator at  $x$ ; so  $\odot x$  is the Declination, and  $rx$  the right Ascension. Draw  $D \odot N^b \parallel EQ$ ; so  $Ds$  is the semidiurnal, and  $Ns$  the seminocturnal Arch, and  $rs$  the ortive Amplitude; for  $D \odot N$  cut the Horizon at  $s$ .

Suppose the Sun's Altitude at any Time of the Day, his Place being  $\odot$ , to be  $39^{\circ}$ . set off  $HK = 39^{\circ}$ . Draw  $\sigma L^b \parallel HR$ , cutting  $D \odot N$  at  $\sigma$ , the Arch  $D\sigma$  reduced to Time, gives the Hour from 12. and drawing  $^c z \sigma N$  through  $z$ ,  $N$ , the Poles of the Horizon, and  $\sigma$ , cutting  $HR$  at  $m$ ,  $Rm$  is the Sun's Azimuth.

To



# 158 Spherique Geometry.

To project the several Circles of the Sphere, on the Plan of the Equinoctial Colure.

Plate VIII.  
FIG. 66.

**T**HE Primitive, Equator, Tropicks and Polars are drawn as before,  $pp$  represents the solstitial Colure and Axis of the World. On  $pp$  produced set off  $66^{\circ}. 30'$ . <sup>a</sup> so you have the Center of the Ecliptick  $E^w Q$ ; on the same  $pp$  set off  $56^{\circ}$ . so <sup>a</sup> you have the Center of  $QHE$  the Horizon of *Edinburgh*. Suppose the Sun's Place, *October 20.* to be  $m. 8^{\circ}$ . that is,  $38^{\circ}$  from  $\omega$  the nearest Equinoctial Point, set off  $Qy = 38^{\circ}$ . and lay a Ruler through  $y$  and  $\omega$ , the Pole of the Ecliptick, that cuts the Ecliptick at  $\odot$  the Sun's Place. Draw  $pop$  cutting the Equator at  $x$ ; so  $Ex$  is the right Ascension, and  $ox$  the Declination; through  $o$  and  $\pi$ , the Pole of  $pop$ , lay a Ruler that cuts off  $Qm =$  Declination; the Point, where a Tangent drawn through  $m$  cuts  $pp$  produced, is the Center of the Parallel of Declination  $mon$ , which cuts the Hori-

ZON

<sup>a</sup> 2 b.

# Spherique Geometry. 159

zon at  $s$ ,  $\sigma$ ; so  $s\sigma$  is the full diurnal Arch, and  $\sigma E$  the ortive or occidial Amplitude.

*To project the Sphere on the Plan of the Horizon of Edinburgh, Latitude  $56^\circ$ .*

Plate VIII.  
FIG. 67.

**D**RAW the North and South Line  $NSLE$  the prime Vertical; the Equator being elevated above the Primitive  $34^\circ$ . the North Semicircle of the Ecliptick  $57^\circ. 30'$ . and the South  $10^\circ. 30'$ . are easily<sup>a</sup> drawn, and<sup>a 2 h.</sup> are here represented by  $uQE$ ,  $uSE$ ,  $u\varpi E$ , the<sup>b</sup> Point  $S$  is one Extremity<sup>b 5 h.</sup> of the Diameter of the Tropick of  $S$ ; from  $z$ , on  $zN$  produced, set off  $7\frac{1}{2} 100^\circ. 30'$ . ( $= zP + PS = 34^\circ + 66^\circ. 30'$ .) that gives the other Extremity of its Diameter. So  $TS$  is easily drawn. The same Way is the Tropick of  $\varpi$  drawn.

Let the Sun's Place be  $8^\circ \gamma$ ,<sup>c</sup> find<sup>c 16 h.</sup>  $\odot$  his Place in the Ecliptick. Draw  $P\odot x$  cutting the Equator at  $x$ ; so  $\odot x$  is the Declination, and  $\gamma x$  the right Ascension. Draw<sup>d</sup> the Parallel<sup>d 14 h.</sup> of

# 160 *Spherique Geometry.*

of Declination  $D \odot d$ ; so  $Dy$  is the semidiurnal Arch,  $ys$  the Meridian Altitude, and  $Du$  the ortive or occidental Amplitude. Take  $r \frac{1}{2}$  Co-altitude for a Radius, and on  $z$ , as a Center, strike an Arch cutting  $D \odot d$  at  $g$ , so  $gy$  is the Hour from 12. and  $cs$  the Azimuth from South.

By Means of this Projection plain Dialing may be performed, for the Hour-lines being nothing else but the common Sections of the Hour-circles with the Dial-plans; having, in this Projection, described the Hour-circles, and the Circle of the Sphere to which the Plan is parallel, their common Sections give the Hour-distances.

## I. *To make an horizontal Dial.*

Plate VIII.  
FIG. 68.

**T**He common Sections of the Hour-circles, with the Horizon, cut off the Hour-distances  $N1$ ,  $N2$ ,  $N3$ , &c. from  $Ns$  the Meridian, or Line of 12.

*Calcul.* In the spherique  $\Delta^{le} NPI$ , right angled at  $N$ ,  $\angle^{le} NPI = 15^\circ$ . the Hour-distances on the Equator,  $NP$   
= Lat.



## Spherique Geometry. 161

= Lat. and R.  $^sNP :: ^tNPI. ^tNI. i. e.$   
 $R. ^sLat. :: ^tH. D. \text{ on the Equator } \cdot$   
 $^tH. D. \text{ on the Horizon.}$

### II. To make a direct erect Vertical Dial.

**T**He common Sections of the Hour-circles, with 6  $\approx$  6 the prime Vertical, cut off  $za, zb, zc,$  &c. the Hour-distances from N S, the Line of 12.

*Calcul.* In the  $\Delta^{le} PZA$ , right angled at  $z$ , the  $\angle^{le} ZPA = 15^\circ$ . and  $PZ =$  Co-lat. and R.  $^sPZ :: ^tZPa. ^tZa, \&c.$

### III. To make an erect declining Vertical Dial, declining, suppose from South, Westward $25^\circ$ .

**S**Et off  $6d = 25^\circ$ . joyn  $Dzd$ , that is the declining Circle. Draw  $zzz \perp Dd$ ; so  $z, z.$  are the Poles of  $Dd$ ; joyn  $zPz$ , cutting  $Dd$  at  $i$ ,  $zPz$  is the Substilar,  $pi$  the Elevation of the Stile, and  $iz$  the Distance of the Substilar from the Meridian, the Ar-  

X

ches

# 162 *Spherique Geometry.*

ches  $z p$ ,  $z q$ ,  $z r$ , &c. give the Forenoon Hour-distances, &c.

<sup>a</sup> 28. Trig.  
P. IV.

*Calcul.* 1. In the  $\Delta^{lc} p i z$ , right angled at  $i$ ,  $p z = \text{Co-lat.}$  and  $\angle^{lc} p z i = \text{Co-decl.}$  and  $R . {}^s p z^a :: {}^s p z i . {}^s p i$ , elev. Stile.

<sup>b</sup> 31.

2.  $\sigma p i . R^b :: \sigma p z . \sigma i z$ . Dist. Substil. from Mer.

<sup>c</sup> 29.

3.  ${}^s p i . R^c :: {}^t i z . {}^t i p z$ .  $i p z + z p p = i p z + 15^\circ = i p p$ . And

4.  $R . {}^s p i :: {}^t i p p . {}^t i p$ , &c.

IV. *To make a declining inclining Vertical Dial, declining  $25^\circ$ . and inclining  $40^\circ$ .*

Plate VIII.  
FIG. 69.

SEt off  $6 d = 25^\circ$ . joyn  $D d$ , draw  $D x d$ , so as the  $\angle^{lc} s d a = 40^\circ$ . So  $D x d$  represents the inclining declining Plan, whose Pole is  $y$ ,  $z z$  the Perpendicular to the Plan,  $a x$  the Distance of the Perpendicular from the Line of 12.  $5 y p m 5$  is the Substilar,  $a m$  its Distance from 12. and  $x m$  its Distance from the Perpendicular,  $d s$  is the Co-declination,  $a b$ ,  $a c$ ,  $a e$ , &c. are the Hour-distances from 12. and  $x a$ ,

# Spherique Geometry. 163

$xa, xb$ , &c. the Hour-distances from the Perpendicular.

In the  $\Delta^{lc} ads$ ,  $R. {}^sds :: {}^{\tau}ads. {}^{\tau}as.$

${}^sas. R :: {}^{\tau}ds. {}^{\tau}das = pam.$

${}^sads. {}^sas :: R. {}^sda.$

$90^{\circ} - da = ax$  Dist. Line 12.  
from the Perpendicular.

In the  $\Delta^{lc} pam$ ,  $R. {}^spa :: {}^{\tau}pam.$   
 ${}^{\tau}pm$ , elev. Stile.

$\sigma pm. R :: \sigma pa. \sigma am$ , Dist. Subst.  
from Mer.

${}^spm. R :: {}^{\tau}am. {}^{\tau}apm$ , Incl. Mer.

In the  $\Delta^{lc} pmb$ ,  $R. {}^spm :: {}^{\tau}mpb.$   
 $({}^{\tau}mpa + apb) {}^{\tau}mb$ , &c.  
for the Hour-distances.









THE  
**A N A L E M M A,**  
 OR  
 ORTHOGRAPHICK PROJEC-  
 TION of the *SPHERE*.

**T**HIS *Projection* supposes the Plate VIII.  
FIG. 70.  
 Eye in the Axis of the Pri-  
 mitive, and at an infinite Di-  
 stance; so that all right Circles, whe-  
 ther lesser or greater, are projected  
 into right Lines, and are measured  
 on the Line of Sines.

Let the Primitive be the solstitial  
 Colure,  $EQ$  the Equator,  $pp$  its Axis,  
 $CL$  the Ecliptick,  $c\odot$ ,  $L\wp$  the Tro-  
 picks,  $AR$ ,  $NT$  the Polars. On the E-  
 cliptick set off  $\gamma\delta = 30^\circ$ . (the Sun's  
 Longitude, *April 11.*) draw  $m\gamma o \parallel$   
 $QE$ , cutting the Horizon  $hr$  at  $s$ ; so  
 $mo$  is the Parallel of Declination  $mQ$   
 $= \gamma d$  the Declination,  $ms$  the semi-  
 diurnal

# 166 *Spherique Geometry.*

diurnal Arch, so the feminocturnal,  $\propto d$  the right Ascension,  $\propto s$  the ascensional Difference,  $s f$  the Distance from due East or West, when rising or setting;  $r f$  his Altitude, when due East or West; and  $r s$  his Amplitude.

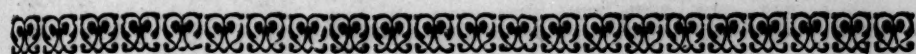
Suppose  $h x$  the Altitude, draw  $x y \parallel h r$ , cutting  $z n$  the prime Vertical, at  $\phi$ , the Parallel of Declination  $m o$ , at  $\sigma$ , and the Circle of 6.  $p p$  at  $\delta$ , then  $\sigma$  shall be the Sun's Place in the *Analemma* at that Time,  $\sigma d$  the Sun's Distance from 6,  $6 \phi$  his Distance from East or West, and  $\sigma r$  his Azimuth from North.

*To reduce any Arch of a lesser Circle, as  $d s$  to the like Arch of a greater.*

Draw  $r o$ , joyn  $s \pi \parallel d r$ , 'tis plain that  $d o . r o :: d s . r \pi$ .







# APPENDIX,

APPLYING

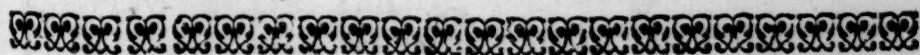
The Doctrine of *Plain Triangles* to taking  
HEIGHTS and DISTANCES,

AND

TO PLAIN and MERCATOR SAILING;

WITH

A short TREATISE of SECTIONS of  
PLANS, in order to the laying out of  
Ground.



# APPENDIX

APPENDING

The Doctrine of Plain Triangles to tak-  
ing Heights and Distances

AND

To PLAIN AND MERCATOR SAILING;

With Treatises of Sections of  
PLANE, in order to the laying out of  
Ground.



# A P P E N D I X,

APPLYING

The Doctrine of *Plain Triangles* to  
taking HEIGHTS and DISTANCES,

AND

To Plain and Mercator SAILING;

WITH

A short Treatise of *Sections* of PLANS,  
in order to the laying out of Ground.

**H**AVING already illustrated the  
Doctrine of spherique Triangles,  
in the Solution of the  
Problems of the Sphere, by Calculation,  
we shall now apply *plain Trigonometry*  
to the taking of Heights and  
Distances, and to plain and mercator  
Sailing.

In order to our laying down on  
Paper the Triangles to be solved, we  
Y must



# 170 APPENDIX.

must first have a Line of Chords, which is already explained.

Plate VIII.  
FIG. 72.

A Line of equal Parts, as  $AB$ , where, if the Divisions from  $c$  to  $B$  be Units, then those from  $c$  to  $A$  shall be Tens; but if the Divisions betwixt  $c$  and  $A$  be reckoned Hundreds, then these from  $c$  to  $B$  shall be Tens, &c. A Line divided in this Sort Sailors call *A Line of Leagues*.

Draw  $cd \perp AB$ , and divide it into Ten equal Parts, through the several Divisions draw Lines  $\parallel AB$ , and complete the Rectangle  $ABGF$ , divide  $FG$ , as  $AB$  is divided, from  $c$ , to the first Division in  $dG$ , draw  $ce$ , and through each Division in  $CB$  draw Lines parallel to  $ce$ , these shall cut off the Units on the parallel Lines  $KI, L2$ , &c. for  $10.10 :: cd.de :: c9.99 :: 9.9$ , &c.

FIG. 73.

A Quadrant is the Fourth Part of a Circle  $DK$  divided into its  $90^\circ$ . and furnished with Sights  $n, o$ , and a Plum-line  $LG$  fixt to the Center  $L$ .

In looking through the Sights of the Quadrant to any Height, the  $\angle^{le} KLG$  (= Arch  $KG$ , cut off by the Plum-

Plum-line  $LG$ ) is equal to the  $\angle^{le} ADF$  formed by Two Rays from the Eye to the Height, one to the Top of the Height, the other parallel to the Horizon. Draw the horizontal Line  $MLE$ , the Plummet  $G$ , falling perpendicular to the Horizon  $MLG$ ,  $= 90^\circ = DLK$ , and  $MLD (= ALE) = MLG - DLG = DLK - DLG = GLK$ .

A *Graphometer* is a Semicircle  $NDB$  FIG. 74 divided into  $180^\circ$ . It has Two Sights,  $x, o$ , on its Diameter, and other Two  $n, z$ , on a Ruler that moves on its Center  $c$ . It is plain, that (the Graphometer being so placed, that any Point  $B$  is seen through the Sights  $o, x$ , and the Ruler being so placed, that any other Point  $A$  may be seen through the Sights  $z, n$ ,) the Arch  $ND$  is the Measure of the Angle  $ACB$ .

## PROB. I.

Plate VIII.  
FIG. 73.

*TO take any accessible Height  $AB$ , on a horizontal Plan.*

In the  $\Delta^{le} ADF$ , right angled at  $F$ ,  $FD = NB$  may be measured.  $\angle^{le} ADF$ , and consequently  $FAD$  is known, and

Y 2

$^s FAD$ .

# 172 APPENDIX.

\* I. Trig.  
P. III.  
° 2.

$^sFAD.FD^a :: ^sFDA.^sFA$ , OR  $R.DF^b ::$   
 $^sADF.AF, AF + FB$ , the Observator's  
Height,  $= AB$ .

Plate VIII.  
FIG. 75.

## PROB. II.

*TO take any inaccessible Height AB,  
or any Part of an inaccessible  
Height, as AC, or CB.*

At any Station D take the Angles  
ADB, CDB, retire to any Distance DE,  
take the Angle AED. So the Angles  
EAD, DAB, BCD, EDA, DCA, are  
known, and

\* I. Trig. ]  
P. III.

$$^sEAD.ED^a :: ^sE.AD.$$

$$R.AD^a :: ^sADB.AB.$$

$$^sACD.AD^a :: ^sADC.AC.$$

$$AB - AC = CB.$$

Plate VIII.  
FIG. 76.

## PROB. III.

*TO find the Distance of Two Places,  
A, B, of which one A is accessible.*

Place your Graphometer at c, and  
take the Angle ACB; find likewise  
the  $\angle^c CAB$ , then  $\angle^c B$  is known,  
measure AC. So  $^sB.AC :: ^sC.AB$ . Q.  
E. F.

PROB.



# APPENDIX. 173

## PROB. IV.

Plate VIII.  
FIG. 77.

**TO find the Distance of Two Places,**  
A, B, both inaccessible.

Choose any Two Stations c, d, measure their Distance cd, with your Graphometer take the  $\angle^{les} ACB, ACD, BDA, BDC$ ; the  $\angle^{les} BCD, CBD, ADC, CAD$ , are known.

In the  $\Delta^{le} BCD$ ,  $^s CBD \cdot CD^2 :: ^s BDC \cdot ^{a} I. Trig. P. III.$

In the  $\Delta^{le} ACD$ ,  $^s CAD \cdot CD :: ^s ADC$ .

In the  $\Delta^{le} ACB$ ,  $BC + CA \cdot BC - CA^b b_3$ .

$:: T \frac{1}{2} CAB + CBA \cdot T \frac{1}{2} CAB - CBA$ .

So the  $\angle^{les} CAB, CBA$ , are easily known.

In the  $\Delta^{le} ACB$ ,  $^s ABC \cdot AC :: ^s ACB$ .

Q. E. F.

The sensible Horizon represents a Circle, whose Plan is the Field on which we stand, or the Surface of the Sea exposed to our View when at Sea; and therefore Sailors, considering no more of this Earth but what they see, have represented the Surface of

of this Earth, as a Plan on which they suppose all the Meridians to be parallel Lines, and the Parallels of Declination, other parallel Lines crossing the Meridians at right Angles. They divide both the Meridians and Parallels of Declination into equal Parts, and these equal Divisions on the Meridians represent the Degrees of Latitude; and the like Divisions on the Parallels of Declination represent the Degrees of Longitude. The Rectangle, formed by these Meridians and Parallels thus drawn, they call the *Plain Chart*, in which it is plain the Degrees of Longitude upon all Parallels are equal. A grand Error.

FIG. 26.

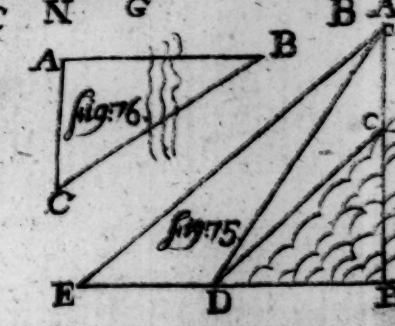
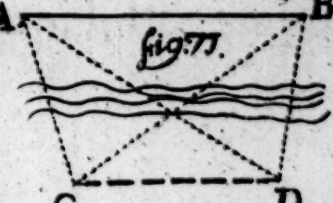
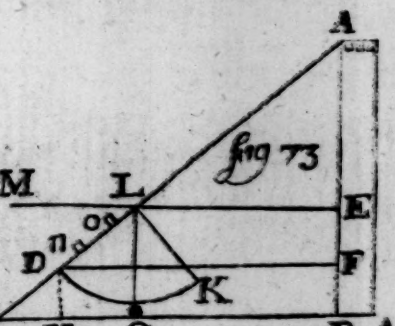
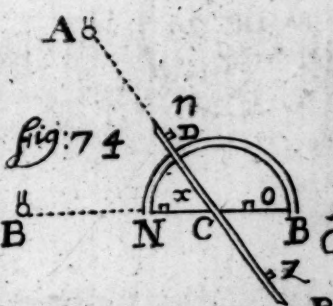
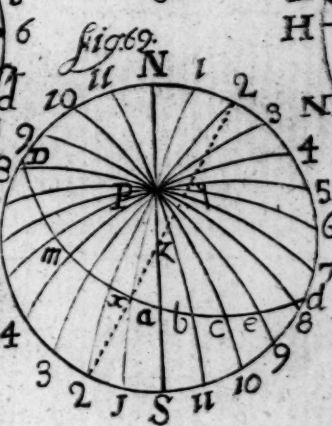
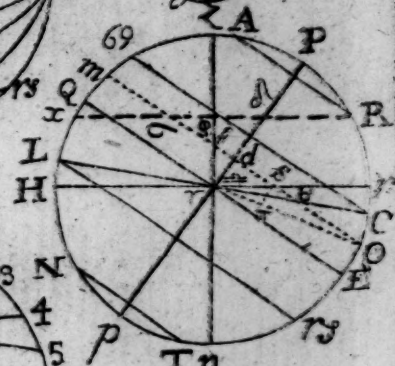
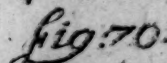
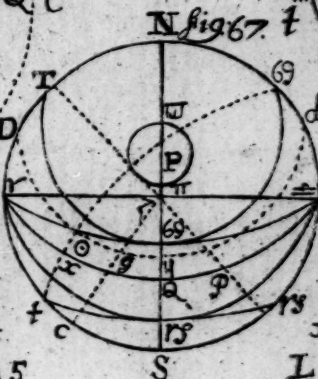
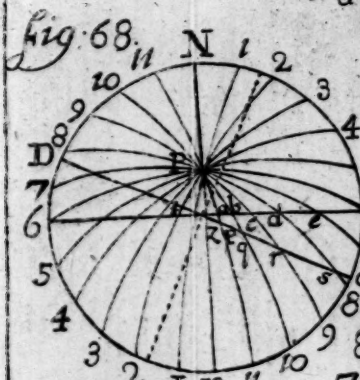
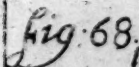
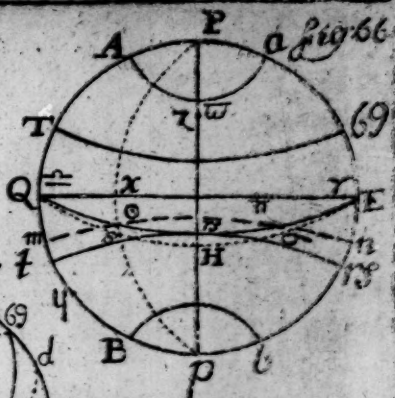
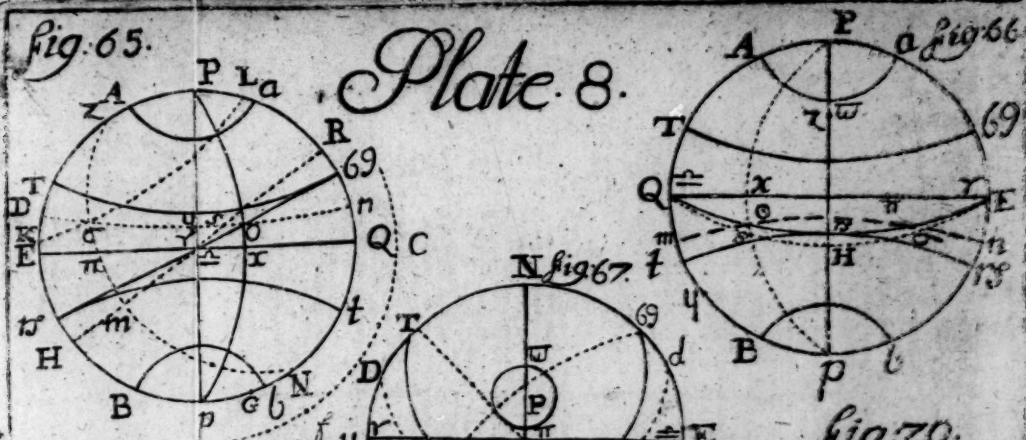
The Sea Compass is a Circle parallel to the sensible Horizon, and is divided into Thirty two equal Parts or Rhumbs, and it is plain these Divisions point out the like Divisions on the sensible Horizon, and therefore the Meridian-line being known by the Needle, the Point of the Compass, on which the Ship steers, *that is*, the Angle that the Ship's Way makes with





Fig: 65.

Plate. 8.



## APPENDIX. 175

with the Meridian, is easily known by reckoning  $11^{\circ}. 15'$ . to one Point:

The Ship failing to any Distance  $NF$ , Plate IX. FIG. 78.  
upon any Point of the Compass, as  $SSE$ , if from  $F$  to  $N$ , the Meridian of the Place at which you set Sail, a Perpendicular be drawn, there shall be formed a right angled  $\Delta^{le} NSF$ , in which the  $\angle^{le} N$  is the Course,  $NF$  the Ship's Way or Distance run,  $NS$  the Difference or Change of Latitude,  $FS$  the Departure: In that Triangle, the Course and Distance run being given, the Difference of Latitude and Departure are easily found by *plain Trigonometry*. For  $R.NF :: ^sF.NS :: ^sN.FS$ . Or thus,

Drawing a Line equal to the Chord of  $90^{\circ}$ . that Line shall represent Eight Points of the Compass; and taking from the Line of Chords, the Degrees answering to the several Points, and Quarter-points, and setting off the Chords of these Degrees on the Line of Eight Points, you shall have made a Line of Rhumbs, by the Help of which Line, and the Line of equal Parts

# 176 APPENDIX.

Parts, called the Line of Leagues, the Triangle may be described, and resolved, thus.

FIG. 79

Suppose we sailed SSE 240 Leagues, draw  $NS$  representing the Meridian of the Place at which you set Sail, upon the Center  $N$ , with the Chord of  $60^\circ$ . strike an Arch  $MD$ , from the Line of Rhumbs take off Two Points equal to SSE, set that from  $M$  to  $D$ , from the Line of Leagues take 240, with that Distance upon the Center  $N$ , strike an Arch  $EP$ , joyn  $ND$ , and continue it to  $P$ , draw  $PS \perp NS$ . So  $NS$  is the Difference of Latitude, and  $PS$  the Departure in Leagues measured on the Line of Leagues.

The same may be done by the Tables of Latitude and Departure, thus.

Find the Distance run in the Margin on the Left-hand, and the Course on the Top or Foot of the Page tracing both, you shall, at the Point of Concourse, find the Difference of Latitude and the Departure.



# APPENDIX. 177

## *To set off any Course.*

Plate IX.  
Fig. 80.

**D**RAW NS at Pleasure, representing the Meridian of the Place at which you set Sail, in it take any Point A for the Place it self, through A draw WE LNS; it is plain, that, making N North, s shall be South, E East, and w West; it is plain too, that all the Points that are compounded of South and East, shall fall betwixt s and E, and all compounded of South and West, betwixt s and w, &c.

## *To draw a Traverse.*

**D**RAW a Line representing the Meridian of the Place where you set Sail at first, and set off the first Course and Distance, parallel to your first Meridian draw the Meridian of the Ship's Place at the Beginning of the Second Course, set off the Second Course, as before, and so on for Third, Fourth, &c. A Line joyning the Points where you at first set Sail, and  
Z the

# 178 APPENDIX.

the Ship's Place, at the End of the last Course, gives the Bearing of the Two Places, and, letting fall a Perpendicular to the first Meridian, that Perpendicular gives the Departure, and cuts off from the Meridian the Difference of Latitude.

FIG. 81:

## Example.

Sailed from *London* in N. Latitude  $50^{\circ}. 30'$ . SSE  $\frac{3}{4}$  E, 220 Leagues, thence S  $\frac{1}{2}$  E  $\frac{1}{2}$  E, 180 Leagues, thence S  $\frac{1}{2}$  W 310 Leagues, thence NE 140, thence S  $\frac{1}{2}$  E 200 Leagues.

Or otherwise, by the Traverse-table, thus.

For each Course and Distance find the Difference of Latitude and Departure by the Tables, the Difference betwixt the Sum of all the Latitudes Northward, and the Sum of all the Latitudes Southward, gives the Difference of Latitude toward the Part of the greater Latitude. The same Way the Difference betwixt the East and West Departures, gives the Departure toward the Part of the greater.

*Points*

# APPENDIX. 179

| Points.         | Course.             | Distance. | Difference of Latitude. |        | Departure. |       |
|-----------------|---------------------|-----------|-------------------------|--------|------------|-------|
|                 |                     |           | SOUTH.                  | NORTH. | EAST.      | WEST. |
| 2 $\frac{3}{4}$ | SSE $\frac{3}{4}$ E | 220       | 188.7                   |        | 113.0      |       |
| 1 $\frac{1}{2}$ | SSE $\frac{1}{2}$ E | 180       | 172.3                   |        | 52.2       |       |
| 1               | SbW                 | 310       | 304.0                   |        |            | 60.5  |
| 4               | NE                  | 140       |                         | 99.0   | 99.0       |       |
| 1               | SbE                 | 200       | 196.2                   |        | 39.0       |       |
|                 |                     |           | 861.2                   |        | 303.2      |       |
|                 |                     |           | 99.0                    |        | 60.5       |       |
|                 |                     |           | 762.2                   |        | 242.7      |       |

*To reduce Leagues to Degrees of Latitude.*

Divide by 20. the Quote gives Degrees, the Remainder multiplied by 3. gives Minutes.

*To reduce Miles to Degrees.*

Divide by 60. the Quote gives Degrees, the Remainder Minutes.

Z 2

To



*To find how many Miles answer to one Degree of any Parallel of Latitude, supposing Sixty Miles to answer to one Degree of the Equator, Say, as R. 60 ::  $\sigma$  Lat. . Number of Miles answering to one Degree of the Parallel of that Latitude.*

Plate IX.  
FIG. 82.

For, making  $DBC$  the Equator, whose Radius  $AB$ , and  $dbc$  the Parallel of Latitude, whose Radius  $Ab$ , making  $BC$   $1^\circ$  of the Equator, it is plain, Circles being similar Figures,  $bc$  is one Degree of the Parallel of Latitude, and  $AB.BC :: Ab.bc$ , *i. e.*  $R.60 :: Ab.bc = 1^\circ$  of the Parallel of Latitude, but  $Ab$  is  $= \sigma$  Lat.

Plate IX.  
FIG. 83.

For, making  $PEp$  the Meridian,  $EQ$  the Diameter of the Equator, and  $la$  the Diameter of any Parallel of Latitude; it is plain  $le$ , the Radius of that Parallel, is the Sine of  $lp$  the Co-latitude. *Ergo, &c.*

Hence, the Parallels of Latitude decrease in the Ratio of the Radius to  $\sigma$  Lat.

The

## APPENDIX. 181

The Parallels of Latitude, reckon-  
ed from the Pole toward the Equator,  
increase in the Ratio of the  $\sigma$ Lat. to  
the Radius, *that is*, (by PART I.  
*Prop. 11. Trig.*) in the Ratio of the  
Radius to the Secant of the Latitude.  
Therefore Mr. *Wright*, considering  
the Error in the plain Chart, where  
the Degrees of Longitude are made  
equal in all Latitudes, has corrected  
the Sea-chart thus. He allows the  
Meridians to be parallel still, and con-  
sequently the Degrees of Longitude to  
be every where equal, as the Degrees  
of Latitude are on the natural Globe,  
but he augments the Degrees of Lati-  
tude on the Sea-chart, in the same  
Ratio that the Degrees of Longitude  
increase on the natural Globe, rec-  
koning from the Pole towards the E-  
quator, *to wit*, in the Ratio of the  
Radius to the Secant of the Latitude,  
and these several Degrees of Latitude  
he expresses in Parts, of which one  
Degree of Longitude contains a cer-  
tain Number, suppose 60. he calcu-  
lates Tables of these Degrees, which  
he calls *Tables of meridional Parts*.

*To*

*To find the meridional Parts answering to any Difference of Latitude.*

I. **I**F the Latitude be reckoned from the Equator North or South, you'll find the meridional Parts answering to it in the Tables.

II. If both Latitudes be on the same Side of the Equator, the meridional Parts answering to the greater Latitude, lessened by those of the lesser Latitude, give the meridional Parts of the Difference of Latitude.

III. If the Two Latitudes be toward contrary Parts, *viz.* one North and the other South, the Sum of their meridional Parts gives the meridional Parts of the Difference of Latitude.

I. *To find the Ship's Place on the Mercator-chart, the Course and Distance being given, suppose a Ship sails SSW  $24^{\circ}$  Leagues, from N. Latitude  $51^{\circ}.30'$ .*

Plate IX.  
FIG. 84.

**D**Raw the  $\Delta^{le} ABC$ , as directed in the plain Chart, find  $AB$  the Difference of Latitude in Degrees, find



## APPENDIX. 183

find the meridional Parts answering to that Difference of Latitude, set off these Parts from A to D, continue AC till it meet the Perpendicular DE at E. So the Point E shall be the Ship's true Place in the Mercator-chart.

II. *Both Latitudes of Two Places A, C, and their Difference of Longitude being given, to find the Course and Distance.*

IN the  $\Delta^{\text{le}} Abc$ ,  $Ab$  represents the proper Diff. Lat.  $bc$  the Departure,  $Ac$  the Distance,  $\angle^{\text{le}} A$  the Course, and  $Ac b$  its Complement. Plate IX.  
FIG. 85.

In the  $\Delta^{\text{le}} ABC$ ,  $AB$  is the meridional Difference of Latitude,  $BC$  the Longitude,  $\angle^{\text{le}} BAC$  the Course, and  $\angle^{\text{le}} C$  its Complement. Reduce the Difference of Longitude  $BC$ , to Miles or Minutes.

$AB.R :: BC.TA$ , the Course. Reduce  $Ab$  to Miles or Minutes, then  $'Ac b.ab :: R.AC$ , the Distance.

III. *Both*

# 184 APPENDIX.

III. *Both Latitudes and the Course given, to find the Distance and Difference of Longitudes.*

${}^sac b . ab :: R . ac$ , the Distance.  
 $R . AB :: {}^tA . BC$ , the Diff. Long.

IV. *Both Latitudes and the Distance given, to find the Course, and Difference of Longitude.*

$ac . R :: ab . \sigma A$ , the Course.  
 $R . AB :: {}^tA . BC$ , the Diff. Long.

V. *One Latitude, the Course and Difference of Longitude given, to find the other Latitude and Distance.*

${}^sA . BC :: {}^sACB . AB$ , the meridional Difference of Latitude, so the other Latitude is easily found, and consequently the proper Difference of Latitude  $ab$ .

${}^sac b . ab :: R . ac$ , the Distance.

If these Cases be well understood, any other that can be proposed will be very easy.

For





# Plate 9

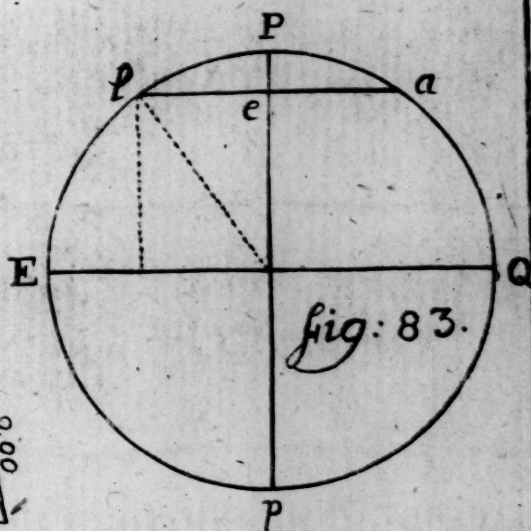
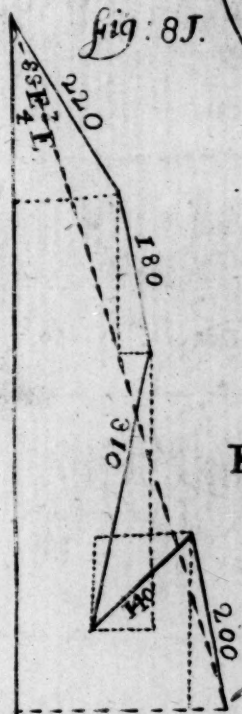
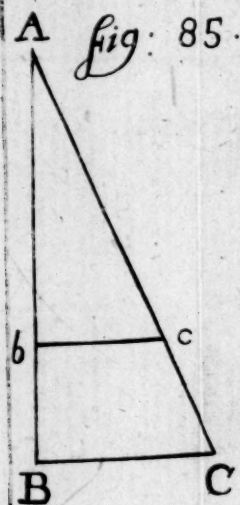
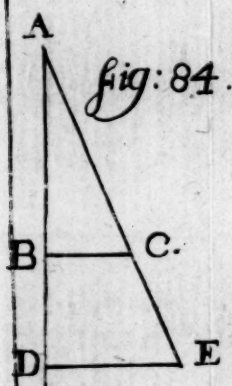
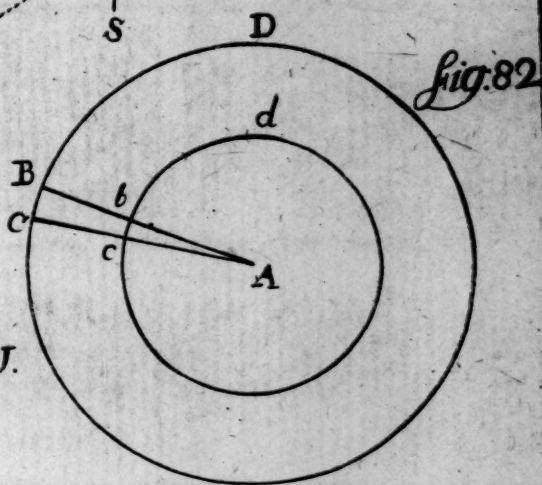
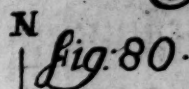
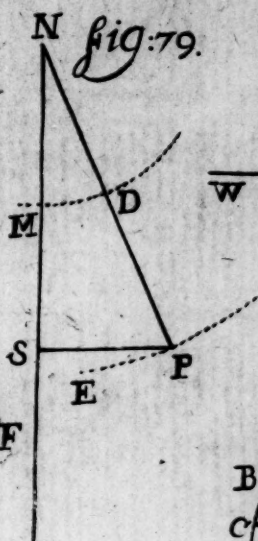
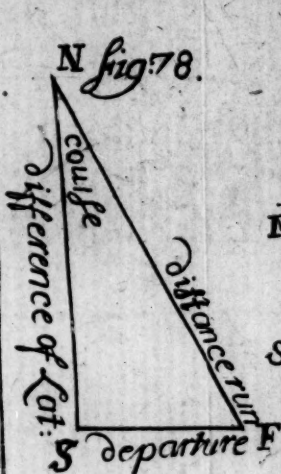


Plate 10.

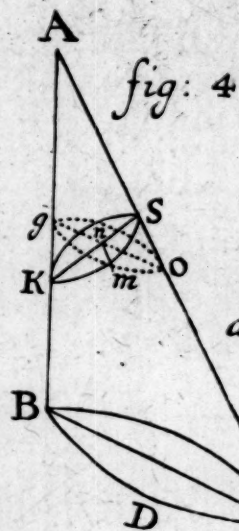
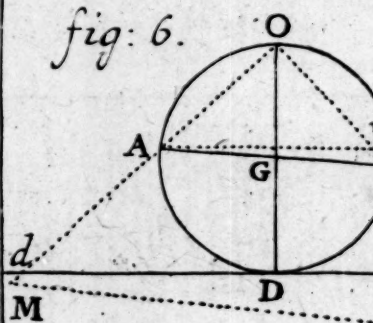
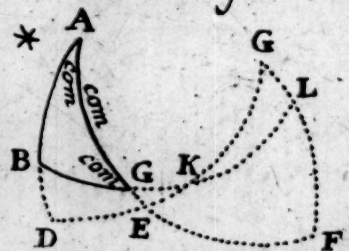


fig: 6.



The 28 followi



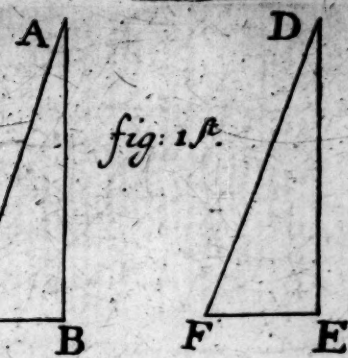


fig: 1<sup>re</sup>.

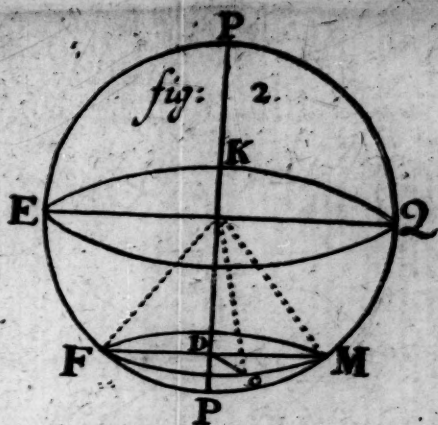


fig: 2.

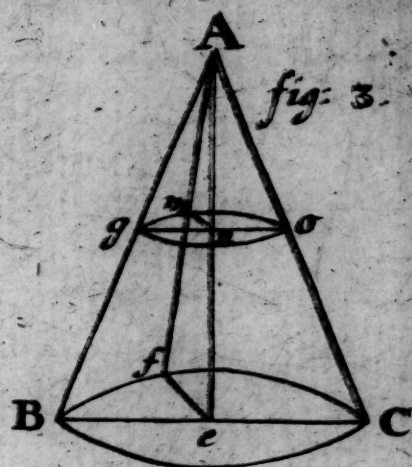


fig: 3.

fig: 4.

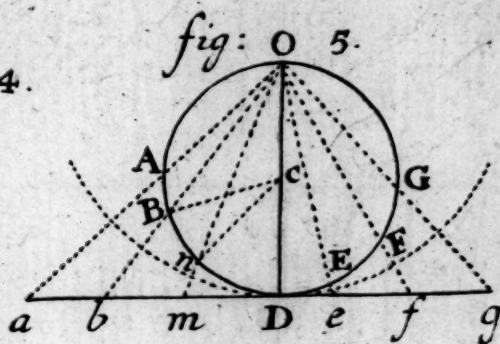


fig: 5.

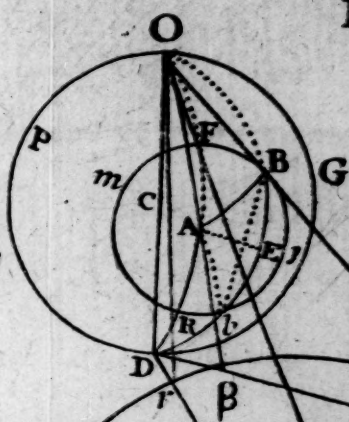


fig: 8.

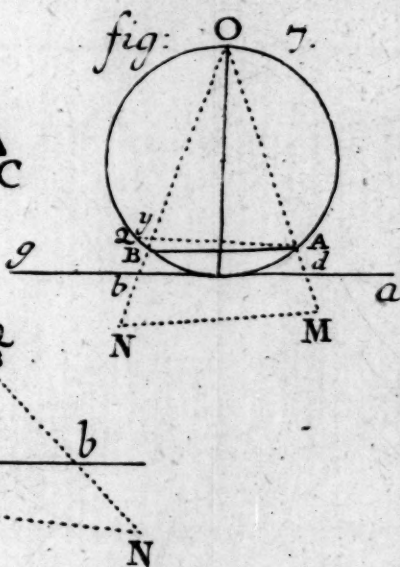
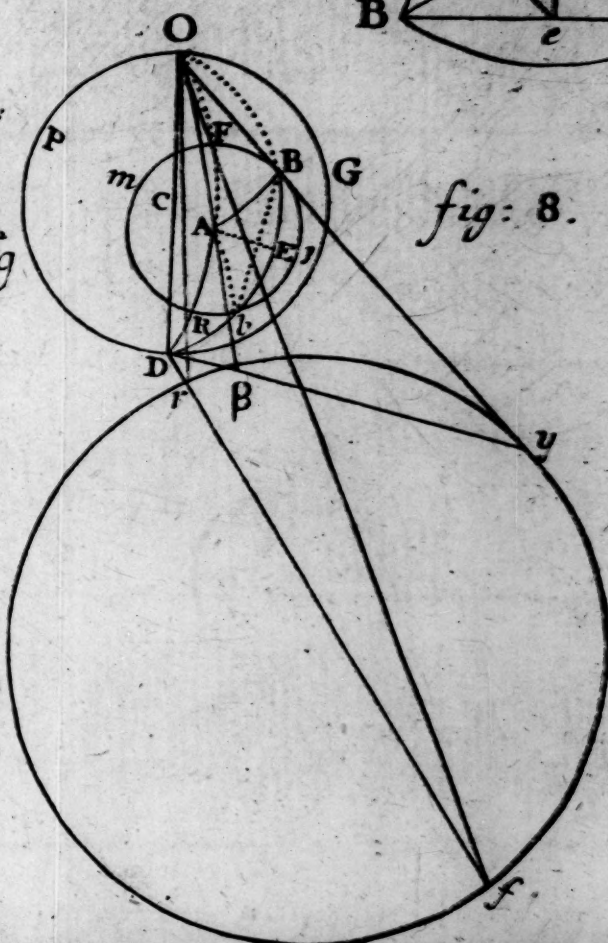
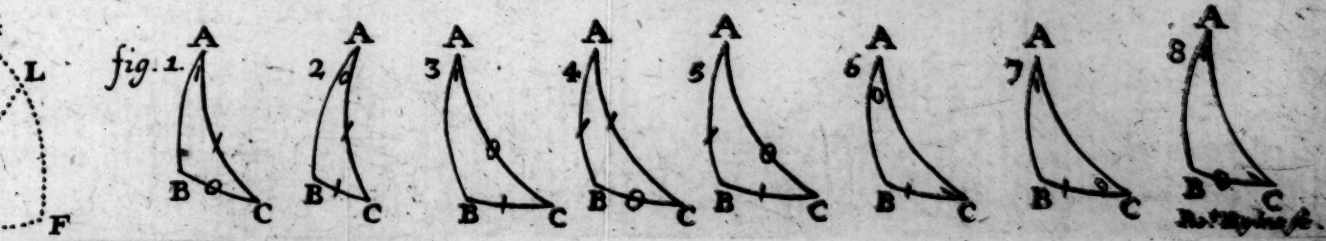


fig: 7.



Following Figures belong to Lord Napier's universal Theorem





## APPENDIX. 185

For oblique Sailing the Two following Examples shall suffice.

I. *Two Ships sail from the same Port A, one sails NbE 120 Miles, to B, the other NNW 188 Miles, to C.*

I demand their Distance, and the Bearing of both Ships from each other.

II. *Two Islands K, M, have the same Longitude, and are distant 38 Miles; a Third Island L is distant from M 60. and from K 80 Miles.*

I demand the Bearings of L from M and K.

The Problems already laid down sufficiently illustrate the Doctrine of plain Triangles, which was all I proposed.



# APPENDIX 187

For opposite bearing the Two fol-  
lowing examples shall suffice.

I. Two ships sail from the same  
port A, one sails N 1/2 E the other  
the other N 1/2 W 188 Miles, so  
I demand their distance; and the  
bearing of both ships from each other.

II. Two Islands A, B are in the same  
longitude, and are distant 80 Miles;  
A Point Island C is distant from A 60  
Miles N 30 E. I demand the bearing of C from  
B and A.

The problems already laid down  
illustrate the solution of  
them. The first which was all I re-



A

A



A

## Short T R E A T I S E

O F

*Sections of P L A N S.*

**T**Is as easy as common to take the superficial Content of any Plot of Ground, in Acres, Roods, &c. by the Graphometer, which we have already described; but, having the superficial Content of a Plot in Acres, &c. to divide that Plot into Parts or Parks, so as each Park may contain an assigned Number of Acres, is perhaps not so common, tho' fully as easy.

In order then to the resolving of this Problem, I shall lay down the following Propositions.

A a 2

P R O P.



Plate XI.  
FIG. 17.

## PROP. I.

*From an Angle A, in a given  $\Delta^{\text{le}}$  ABC, to divide that Triangle in a given Ratio, viz. in the Ratio of G to F, so as the lesser Part may ly toward a given Side AB.*

Divide the Base BC at D, so as BD.  
<sup>a</sup> 10. e. 6.  $DC^a :: G.F.$  Draw AD, and 'tis plain  
<sup>b</sup> 1. e. 6.  $ABD.ADC^b :: BD.DC :: G.F.$  Q.  
*E. F.*

*Schol.* In Practice, thus.

Let the  $\Delta^{\text{le}}$  ABC contain 8 Acres, and the Base 40 of whatever Measure, to divide the Triangle into Two Parts, so as the lesser may be 3. and the greater 5 Acres: Say, as  $8.40 :: 3.15$ , the Base of the lesser Part. Set off 15 from B to D, joyn AD, and it is done.

Plate XI.  
FIG. 18.

## PROP. II. PROB. II.

*From a Point E, in the Side of a  $\Delta^{\text{le}}$  ABC, to draw a Line dividing the Triangle in a given Ratio of K to L, and to lay the lesser Part toward a given  $\angle^{\text{le}}$  B.*

Draw

# APPENDIX. 189

Draw AE, with the Line AD<sup>a</sup> di-<sup>a</sup> 1 b.  
vide the Triangle in the Ratio of K  
to L. Draw DF || AE, joyn EF, and it  
is done.

For FDA<sup>b</sup> = FDE, add to both BDF. <sup>b</sup> 37. e. 1.  
So BFE = ABD, therefore AFEC<sup>c</sup> = <sup>c</sup> Ax. 2.  
ADC, consequently BFE.AFEC :: ABD.  
ADC :: K.L. Q. E. F.

*Praxis.* Find the Point D, as in the  
preceeding Praxis, draw DF || AE, &c.

## PROP. III. PROB. III.

Plate XI.  
FIG. 19.

*TO divide a  $\Delta^{lc}$  ABC, in a given Ra-  
tio of K to I, by a Line parallel to  
one of its Sides.*

Make BE.EC :: I.K, and find BF a  
Mean proportional betwixt BE and BC,  
draw FH || AC, and it is done.

For  $\frac{K+I}{I} = \frac{EC+BE}{BE} = \frac{BC}{BE} = \frac{BC}{BF}$  twice <sup>a</sup> Construct.  
<sup>b</sup> =  $\Delta^{lc} \frac{BCA}{BFH}$  therefore K.I<sup>c</sup> :: BCA — <sup>b</sup> 19. e. 6.  
BFH.BFH :: ACFH.BFH. Q. E. D. <sup>c</sup> 17. e. 5.

## PROP. IV. PROB. IV.

Plate XI.  
FIG. 20.

*TO find Two Lines CF, DF, equal  
in Power to a given Line CD, and  
in Power in a given Ratio of A to B.*  
Divide

# 190 APPENDIX.

<sup>a</sup> 10. e. 6. Divide <sup>a</sup> CD in the Ratio of A to  
<sup>b</sup> 13. e. 6. B, at E <sup>b</sup> find EF a Mean proportional  
 betwixt CE and ED, draw CF, DF, and  
 it is done.

<sup>c</sup> 47. e. 1. For  $CF_q + FD_q^c = CD_q$ , and  $CF$ .  
<sup>d</sup> 4. e. 6.  $FD^d :: CE.EF$ , therefore  $CF_q.FD_q^c ::$   
<sup>e</sup> 22. e. 6.  $CE_q.EF_q^f :: CE.ED^g :: A.B.$  Q.  
<sup>f</sup> 20. e. 6.  $E.D.$   
<sup>g</sup> Hyp.

Plate XI.  
 FIG. 20.

## PROP. V. PROB. V.

*TO divide a Line CD in Power in  
 the given Ratio of B to A.*

Divide CD in the Ratio of B to A,  
 at E, betwixt CE and ED, find a Mean  
 proportional EF, draw CF, DF, con-  
 tinue CF to H, so as  $FH = FD$ , joyn  
 HD, and draw  $FK \parallel DH$ : I say  $CK_q$ .  
 $KD_q :: B.A.$

<sup>a</sup> 22. e. 6. For  $CK_q.KD_q^a :: CF_q.FH_q :: CF_q$ .  
<sup>b</sup> 20. e. 6.  $FD_q^a :: CE_q.EF_q^b :: EC.ED^c :: B.A.$   
<sup>c</sup> Construct. Q. E. D.

Plate XI.  
 FIG. 21.

## PROP. VI. PROB. VI.

*TO diminish a given Plot ABCDE, in  
 the given Ratio of M to N.*

<sup>a</sup> 10. e. 6. Divide AB in F <sup>a</sup>, so as  $AF.AB :: N$   
 $.M$ , betwixt AB, AF, find a Mean pro-  
 portional



# APPENDIX. 191

portional  $Ab$ , draw  $Ac$ ,  $Ad$ , through  $b$ , draw  $bc \parallel Bc$ , draw  $cd \parallel CD$ , &c. it is plain  $\frac{ABCDEb}{abede} = \frac{AB}{ab}$  twice,  $= \frac{AB}{AF} =$  <sup>b</sup> 20. e. 6.

$\frac{M}{N}$ . Q. E. D.

Schol. In the same Manner, to augment a Plot  $Abcde$ , in a given Ratio of  $N$  to  $M$ , make, as  $N.M :: Ab.Af$ , betwixt  $Ab$  and  $Af$  find a Mean  $AB$ , &c.

Plate XI:  
FIG. 22.

## PROP. VII. PROB. VII.

TO make a Triangle which shall contain any given Number of Acres (suppose 5 Acres, 2 Roods, 30 Falls) and whose Base shall be any given Number, suppose 50 Falls.

Double of 5 Acres, 2 Roods, 30 Falls, reduced to Falls, gives 1820 Falls, for the Area of a right angled Parallelogram, which divided by 50. gives  $36\frac{2}{3}$ . for the Altitude of that Parallelogram; describe that Parallelogram  $AD$ , draw  $AE$ ,  $BE$ , and it is done, as is plain from 41. e. 1.

PROP.

# 192 APPENDIX.

## PROP. VIII. PROB. VIII.

Plate XII.  
FIG. 1.

*TO reduce a Trapezium ABCD to a Triangle, by a Line drawn from one of its Angle B.*

Draw the Diagonal BD, through c, draw CE  $\parallel$  BD, meeting AD at E, joyn BE: I say, the  $\Delta^{lc}$  ABE = ABCD.

\* 37. c. 1.

For the  $\Delta^{lc}$  BDE<sup>a</sup> = BDC, take BDM from both, and DME = BMC, add ABMD to both, and ABE = ABCD. Q. E. F.

Plate XII.  
FIG. 2.

## PROP. IX. PROB. IX.

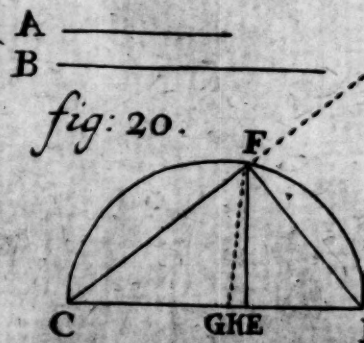
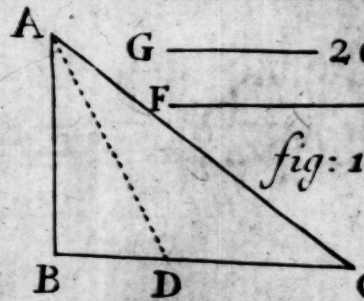
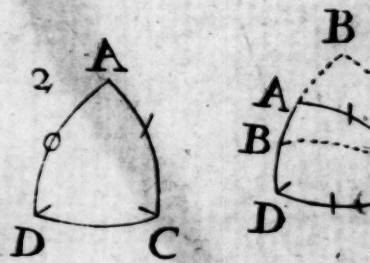
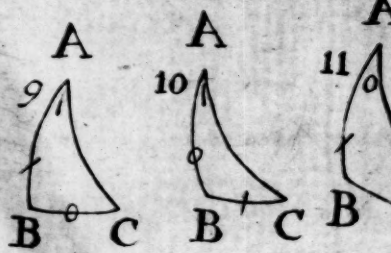
*TO reduce a Trapezium ACDB to a Triangle, by Lines drawn from a Point H in the Side of the Trapezium.*

Draw HC, HD, through A and B, draw AE  $\parallel$  HC, and BF  $\parallel$  HD, meeting with CD continued at E and F, joyn HE: I say, HEF = ACDB.

For the  $\Delta^{lc}$  AEH = AEC, take AEM from both, and MEC = AMH, add HMC to both, and HEC = ACGH. In like Manner HGF = HGDB, therefore HEF = ACDB. Q. E. F.

PROP.

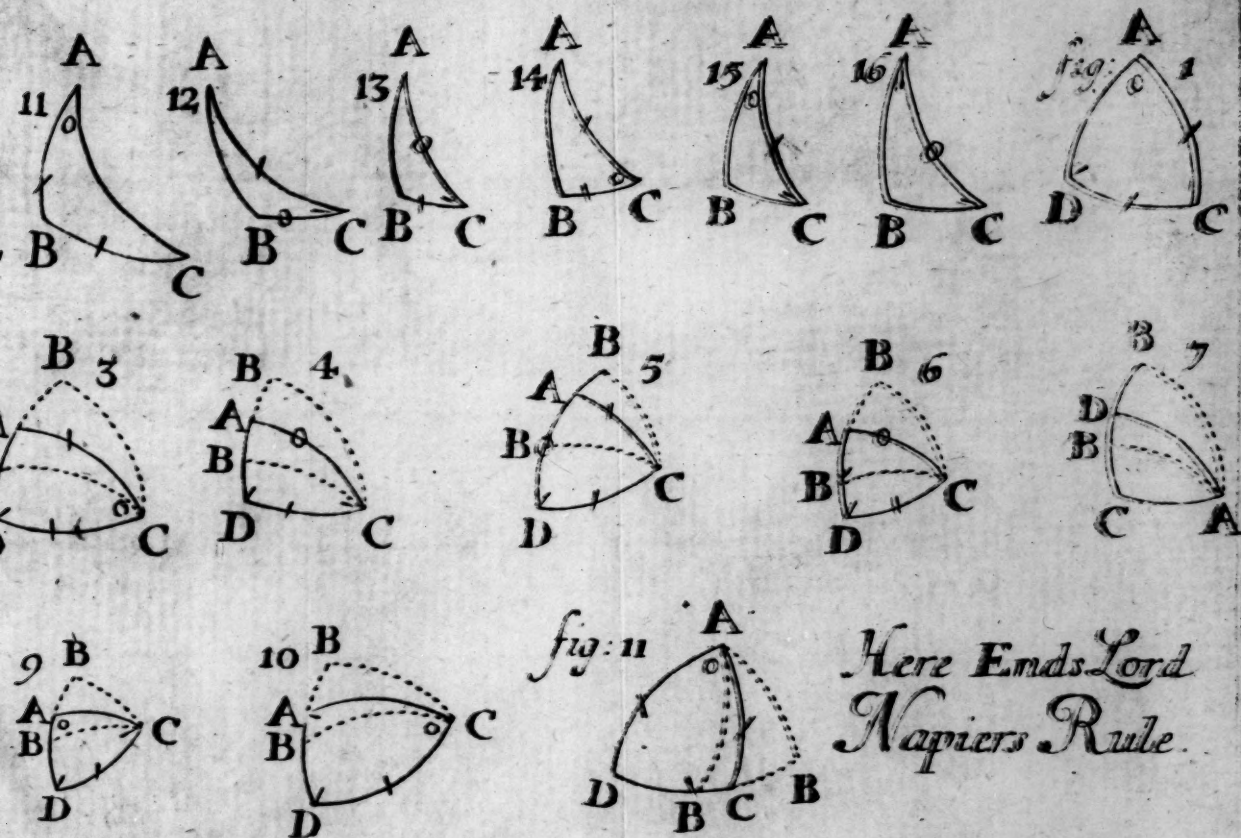
# The Cont



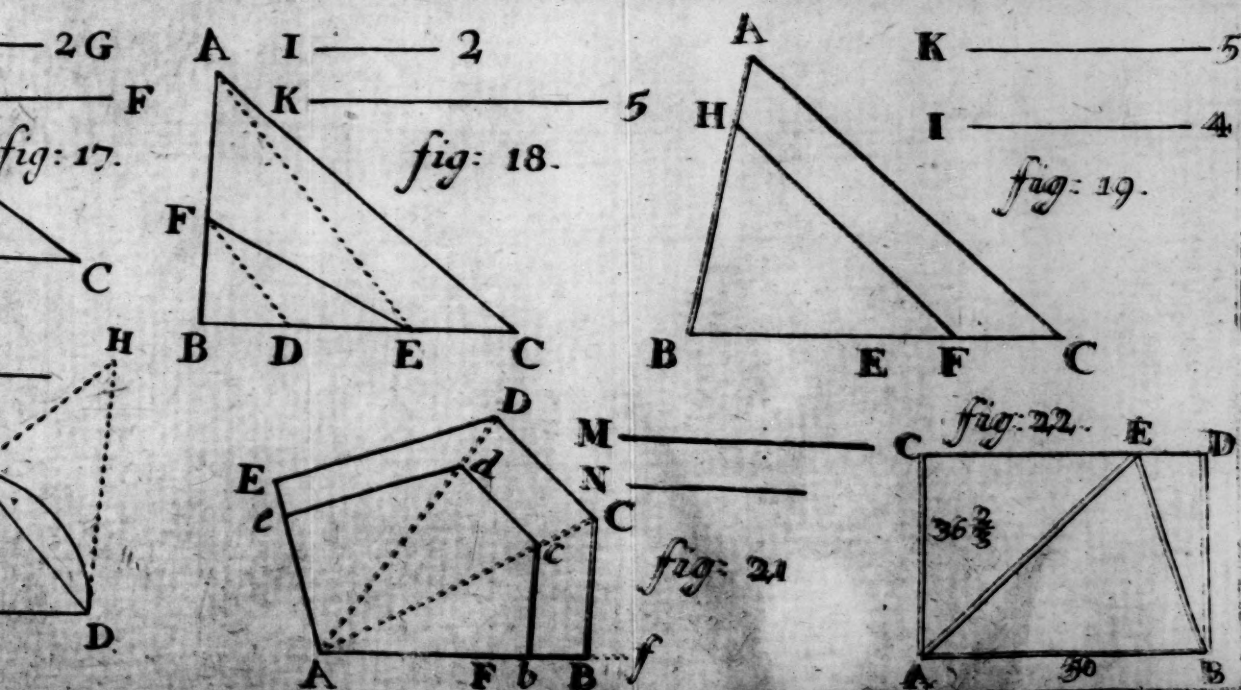
Re: Myline sc.



# Continuation of Lord Napiers Rule. Plate 11.



Here Ends Lord  
Napier's Rule.



# APPENDIX. 193

## PROP. X. PROB. X.

Plate XII.  
FIG. 3.

*TO reduce an irregular Pentagon  
ABCDE to a Triangle, by Lines  
drawn from one of its Angles c.*

Draw CA, CE, through B and D,  
draw BF  $\parallel$  CA, and DG  $\parallel$  CE, joyn CF,  
CG: I say, CFG = ABCDE.

For BFA = BFC, therefore mFA  
= BmC, add to both cmAE, and  
CFE = CBAE.

Likewise DGE = DGC, take DnG  
from both, and Gne = Dnc, add  
cne to both, and CGE = CDE. Er-  
go, &c. Q. E. D.

## PROP. XI. PROB. XI.

Plate XII.  
FIG. 4.

*TO reduce an irregular Figure of a-  
ny Number of Sides, as ABCDEFG,  
to a Triangle.*

Draw BD and GE, through c and f,  
draw CK  $\parallel$  BD, and FL  $\parallel$  GE, joyn BK  
and GL; it is plain BDK = BDC, take  
BDS from both, and DKS = BCS.

The same Way GEL = GEF, take  
GER from both, and ERL = GRF.

B b

In-

# 194 APPENDIX.

In stead of the  $\Delta^{les} DSK, ErL$ , which are cut off from the Figure by the Lines  $BK, GL$ , substitute their Equals  $BCS, GFr$ , so we have a Pentagon  $ABKLG$ , equal to the Figure  $ABCDEFG$ , and that Pentagon may be reduced to a Triangle, by the preceeding Problem. *Q. E. F.*

Plate XII.  
FIG. 5.

## PROP. XII. PROB. XII.

*From an Angle B of a given Trapezium ABCD, to draw a Line dividing the Trapezium in a given Ratio of G to H.*

Resolve the Trapezium into the  $\Delta^{les} ABF$ , make  $AE.EF :: G.H$ , draw  $BE$ , and it is done.

For  $ABE . EBCD :: ABE . EBF :: AE.EF :: G.H$ . *Q. E. F.*

Plate XII.  
FIG. 6.

## PROP. XIII. PROB. XIII.

*From a Point H, in the Side of a Trapezium ABDC, to draw a Line dividing the Trapezium in a given Ratio of O to Q.*

Re



# APPENDIX. 195

Resolve the Trapezium into the  $\Delta^{\text{le}}$  HEF, make  $EG.GF :: O.Q$ , joyn HG, and it is done.

For AHGC.GHBD :: HGE.HGF :: EG.GF :: O.Q. Q. E. F.

## PROP. XIV. PROB. XIV.

Plate XII.  
FIG. 7.

*From an Angle B, in a given Figure ABCDE, to draw a Line dividing the Figure in a given Ratio of R to S.*

Resolve the Figure into the  $\Delta^{\text{le}}$  BFG, make  $FH.HG :: R.S$ , draw BH, and it is done.

For ABHE.HBCD :: BFH.BGH :: FH.GH :: R.S. Q. E. F.

## PROP. XV. PROB. XV.

Plate XII.  
FIG. 8.

*From a Point D, in the Side of a given Triangle ABC, to draw Lines dividing that Triangle into a given Number of equal Parts, suppose Five.*

From D, to the opposite Angle A, draw DA, <sup>a</sup> divide BC into Five equal Parts at E, F, G, I, draw <sup>b</sup> EN, FM, <sup>b</sup> 31. c. 1. Ec. || AD, joyn DN, DM, DL, DK, and it is done.

B b 2

For

# 196 APPENDIX.

<sup>c</sup> 2. e. 6. For because  $BE \cdot EF^c :: BN \cdot NM$ ,  
<sup>d</sup> 14. e. 5. and  $BE = EF$ , therefore  $BN^d = NM$ ,  
and the same Way  $MN = ML$ , there-  
<sup>e</sup> 38. e. 1. fore the  $\Delta^{le} DBN^c = DNM^c = DML$ .

In the Trapezium  $AKDM$ , the Dia-  
gonal  $AD$  being drawn, and  $IK$  paral-  
<sup>f</sup> 74.  $el$  to it, meeting  $BA$  continued at  $O$ ;  
it is plain, the  $\Delta^{le} MDO^f = AKDM$ ,  
and  $DML \cdot DKAL :: DML \cdot DLO :: ML$   
 $\cdot LO$ . But, (because  $FG \cdot GI^c :: ML$   
 $\cdot LO$ . and  $FG = GI$ )  $ML = LO$ , there-  
fore  $AKDL = DML$ .

After the same Manner we may  
shew, that  $DK$  bisects the Trapezium  
 $ACDL$ . *Q. E. F.*

Plate XIII:  
FIG. I.

## PROP. XVI. PROB. XVI.

*From a Point G, in the Side of a  
Polygon  $ABCDEF$ , to draw a Line  
dividing the Polygon in the given Ra-  
tio of  $R$  to  $S$ .*

Draw  $IK$  resolving the Figure into  
Two lesser Polygons  $ABCF$ ,  $EDCF$ ,  
<sup>a</sup> find the  $\Delta^{le} GIK = ABCF$ , and by  
<sup>b</sup> 8, 9, 10 *h.* that Means draw  $GN$  dividing  $ABCF$   
in the Ratio of  $R$  to  $S$ . After the  
same Manner find the  $\Delta^{le} NLM =$   
 $EDCF$ ,

A  
B

F

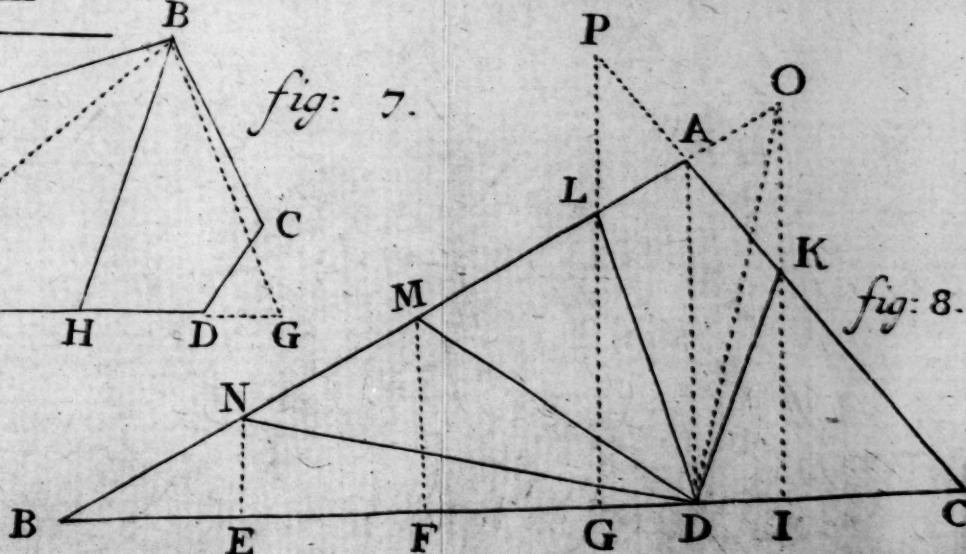
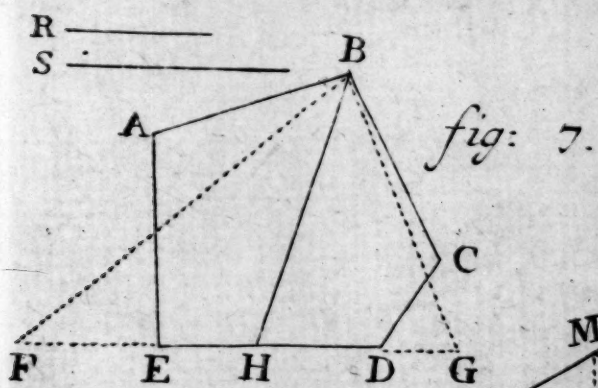
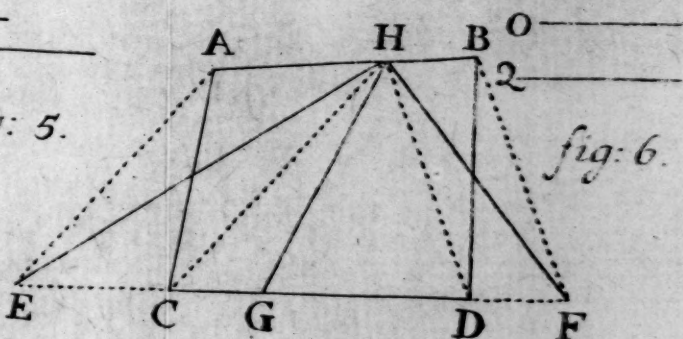
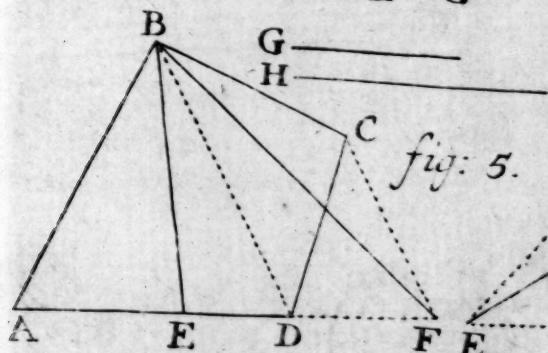
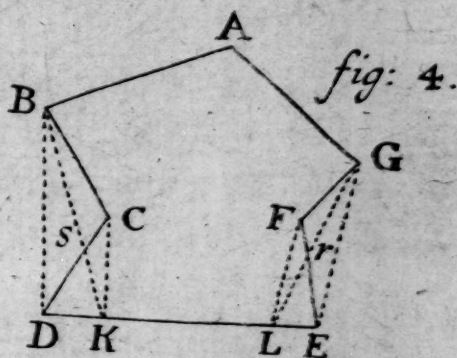
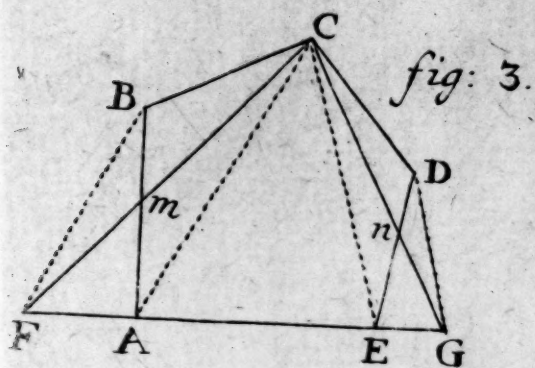
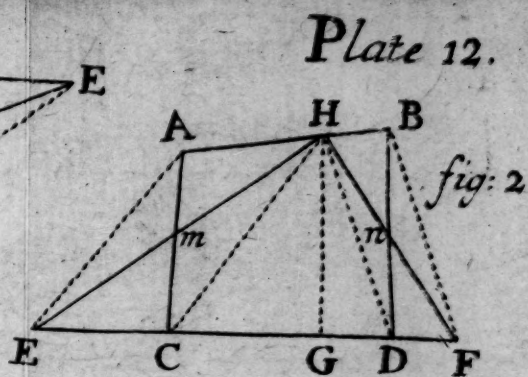
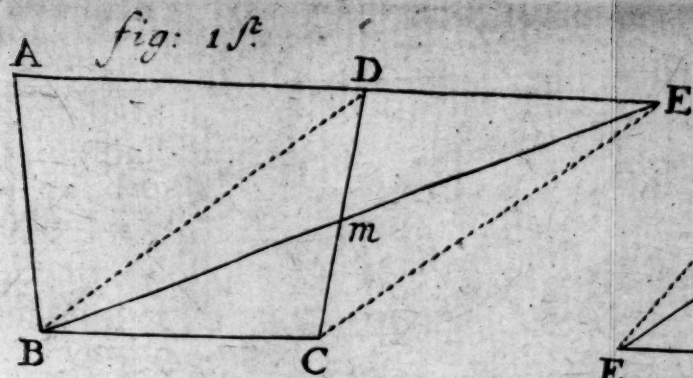
A

R -  
S -

F

A





# APPENDIX. 197

EDCF, and draw NO dividing EDCF in the Ratio of R to S. So GBCDON. AGNEOF :: R. S, joyn GO, through N draw ND || GO, joyn GD: I say, GBCD.GAFED :: GBCDON.AGNOEF :: R.S.

For the Line GD cuts off from AGNOEF the  $\Delta^{\text{le}}$  GNQ, and from GBCDON the  $\Delta^{\text{le}}$  DOQ. Now DGN<sup>b</sup> 37. e. i. = DON, and taking DQN from both, there remains DOQ = GNQ. *Ergo*, &c. Q. E. F.

## PROP. XVII. PROB. XVII.

Plate XIII.  
FIG. 2.

*From a Plot of Ground ABCDEFGIK, given in Acres, &c. to cut off a Part containing a given Number of Acres, Roods, &c.*

Let the Plot contain 11 Acres, 1 Rood, 34 Falls, 13.5 Feet, or 627700 Feet, from which it is required to cut off a Part of 4 Arces, 3 Falls, 109.05 Feet, or 231398 Feet, by a Line drawn from the Angle G.

From G, to any other Angle, draw a Line which you conjecture may cut off a Part containing near the Measure

# 198 APPENDIX

sure demanded, as  $GD$ , find the Content of the Part cut off,  $GFED = 225360$  Feet, that is short of the Part demanded, *viz.*  $231398$  by  $16038$  Feet, find  $GD$  in Feet, which is  $700$ , by Half that, *viz.*  $350$ , divide  $16038$ , draw  $LM \parallel DG$ , having its Distance from  $DG = 45 \frac{188}{350}$  the Quote, draw  $GL$ , and it is done.

For the Altitude of the  $\Delta^c GDL$ , is  $45 \frac{188}{350}$ , which multiplied into  $\frac{1}{2} DG$ , gives  $16038$ , the Excess of the Demand above  $GFED$ , therefore  $GFEDL$  is the Part sought. *Q. E. F.*







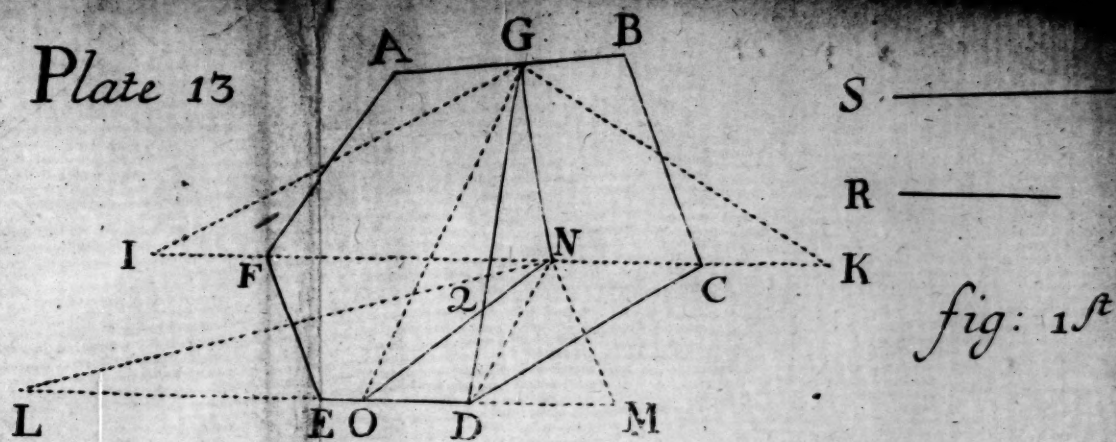


fig: 1<sup>st</sup>

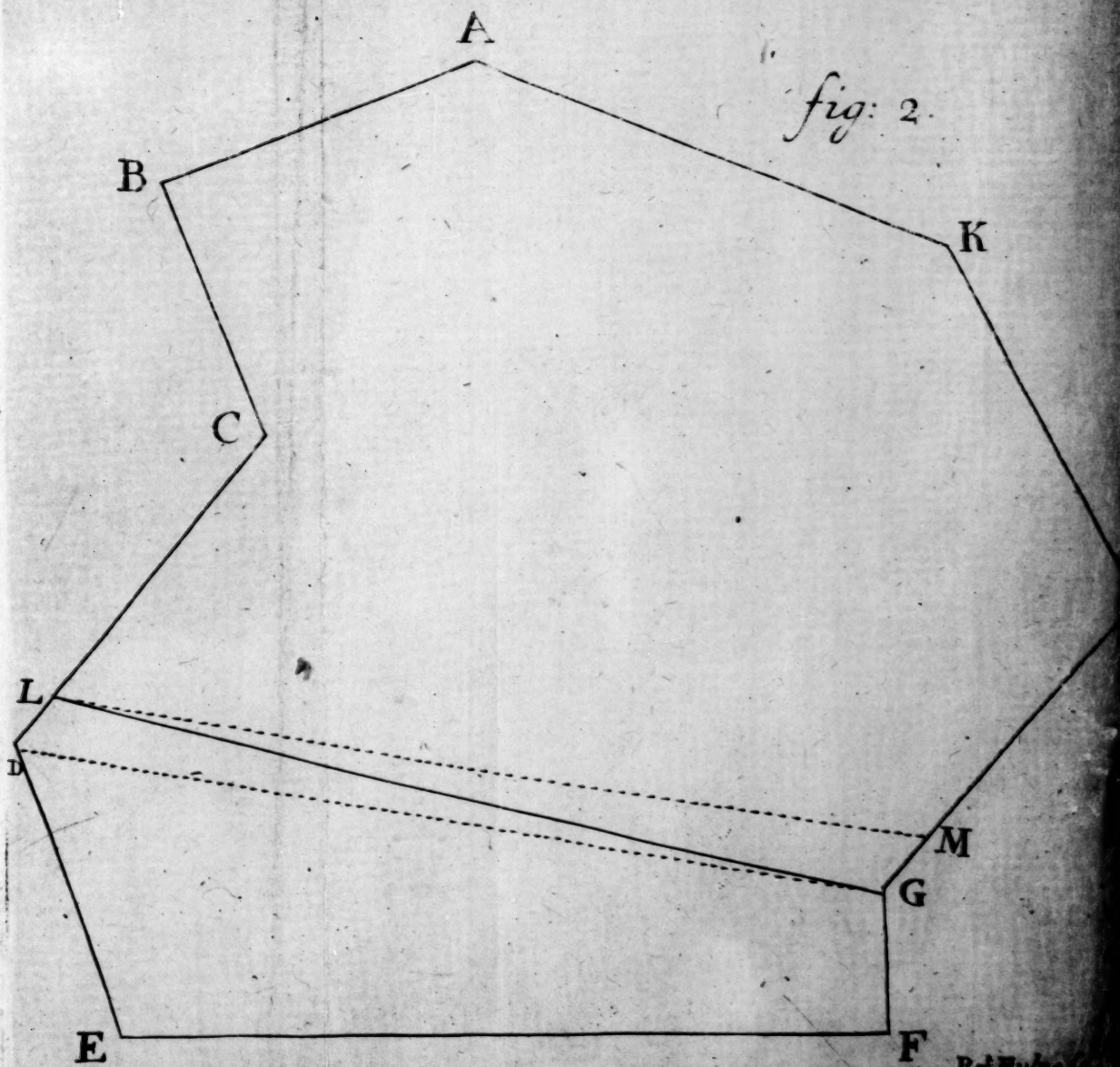


fig: 2



A  
SHORT INTRODUCTION  
TO  
ARCHITECTURE,  
BEING  
A brief *Explication* of the Five ORDERS,  
ACCORDING TO  
*PALLADIO* and *SCAMOZZI*.





A  
SHORT INTRODUCTION  
TO  
ARCHITECTURE  
BEING  
A Brief Explanation of the Five Orders





A

SHORT INTRODUCTION

TO

ARCHITECTURE,

BEING

A brief Explication of the Five *Orders*,

ACCORDING TO

*PALLADIO* and *SCAMOZZI*.

**A**RCHITECTURE is the Art of building well, and has for its Object the Strength, Beauty and Conveniency of the Building.

A *Column* or *Pillar* is whatever supports any Weight or Burden.

A *Male Statue*, supplying the Place of a Pillar, is called *An Atlas*.

*Female Statues* are called *Cariatides* or *Carians*.

C c

Order

## 202 *A short Introduction*

*Order* is an Ornament in Architecture, consisting of a Pillar having a Base for its Support, and a *Travaison* representing the Weight or Burden that it bears.

The whole Face of the Building that is beautified with various Ornaments, is called *Ordonnance* or *Colonnade*; the First Term is taken from the several Orders of Architecture that adorn it, and the Second from the Column or Pillar, which is the Rule and Measure of the rest.

Its Parts are Four.

Plate XIV.

1. A, the Column or Pillar,
2. B, the *Pedestal* or *Stylobate*,
3. C, the *Entablement* or *Travaison*,
4. N, the *Frontispiece*.

Again, each of these Parts has Three Members.

These of the Column are,

1. E, the *Fust* or *Trunk*,
2. D, the *Capital*,
3. F, the *Base*.

These of the Pedestal,

1. L, the *Dy*,
2. M, the *Cornice*,

3. K, the



3. K, the Socle or Base of the Pedestal.

The Entablement,

1. G, the Architrave,

2. H, the Frize,

3. I, the Cornice.

The Frontispiece,

1. P, the Tympan,

2. O, the Cornice,

3. Q, the Acroteres.

The several Parts of every Order are composed of many lesser Parts, which are either plain, convex, concave, or convexo-concave, and are called *Members* or *Moulures*.

## DEFINITIONS.

I. *Projection* or *Projecture* is the Excess of Latitude of one Part above that of another.

II. *Reglet*, *Filet*, or *Listeau* is a <sup>Plate XV.</sup> small Member every where plain. FIG. 1.

III. *Tore*, *Baton*, or *Bozel* is a large Member with a semicircular Convexity. FIG. 2.

IV. *Astragal* differs from the *Tore* in Magnitude only.

C c 2

V. *Echin*,

## 204 *A short Introduction*

FIG. 3. V. *Echin*, *Ove*, or *Œuf* is a Member having a Convexity less than a Semicircle, and its Projection  $BC = \frac{2}{3}BD$ , its Altitude  $= Bb$ .

FIG. 4. VI. *Cymaise Dorick* or *Cavett* has a Concavity less than a Semicircle, and its Projection  $AE = Ad = \frac{1}{2}AB$  its Altitude.

FIG. 5. VII. *Scotie* or *Trochile* is a large Member, whose Concavity  $CDK$  is composed of Two Arches.

FIG. 6. VIII. *Cymaise Lesbian*, or *Talon* is a concavo-convex Member  $LMN$ , whose Projection  $OL = \frac{1}{2}ON$  its Altitude,  $= oa$ .

FIG. 7. IX. *Sima*, *Doucine*, or *Gueule* is a concavo-convex Member  $CFB$ , whose Projection  $AC = AB$  its Altitude.

FIG. 8. X. *Apophygis* or *Chanfrain* is a concave Member  $AC$ , joyning a plain Member with any other Member whether plain or curved.

Plate XV.  
FIG. 2.

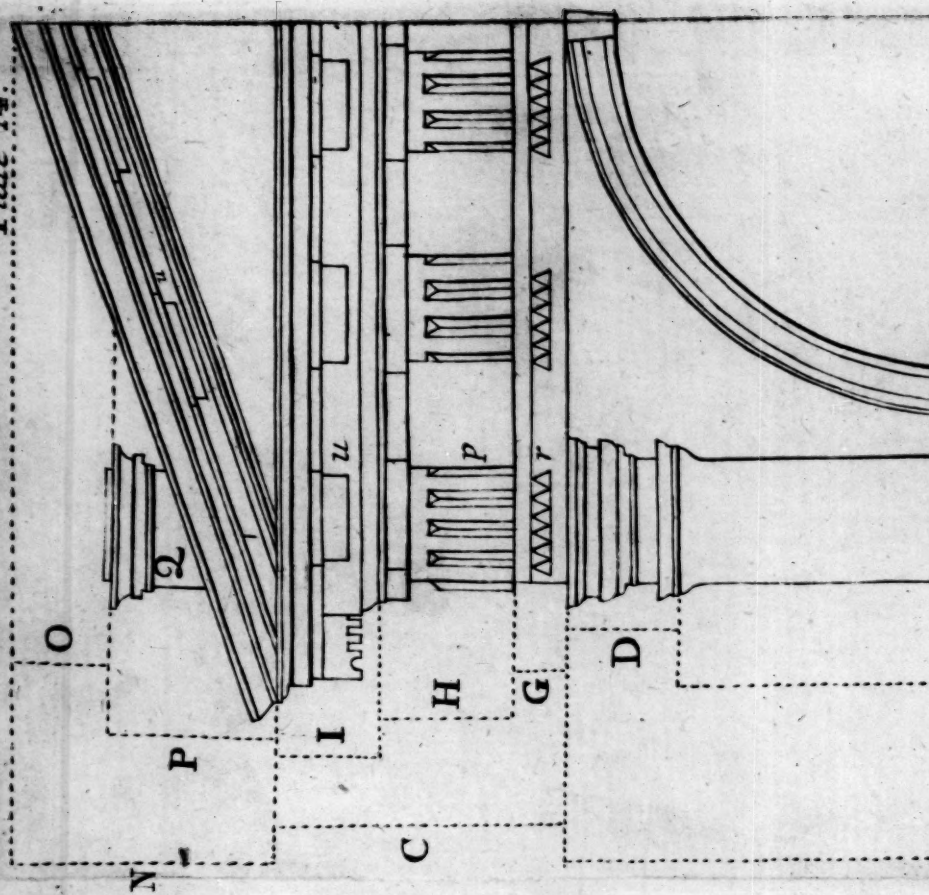
### PROB. I.

**T**O describe a *Tore* or *Astragal*.

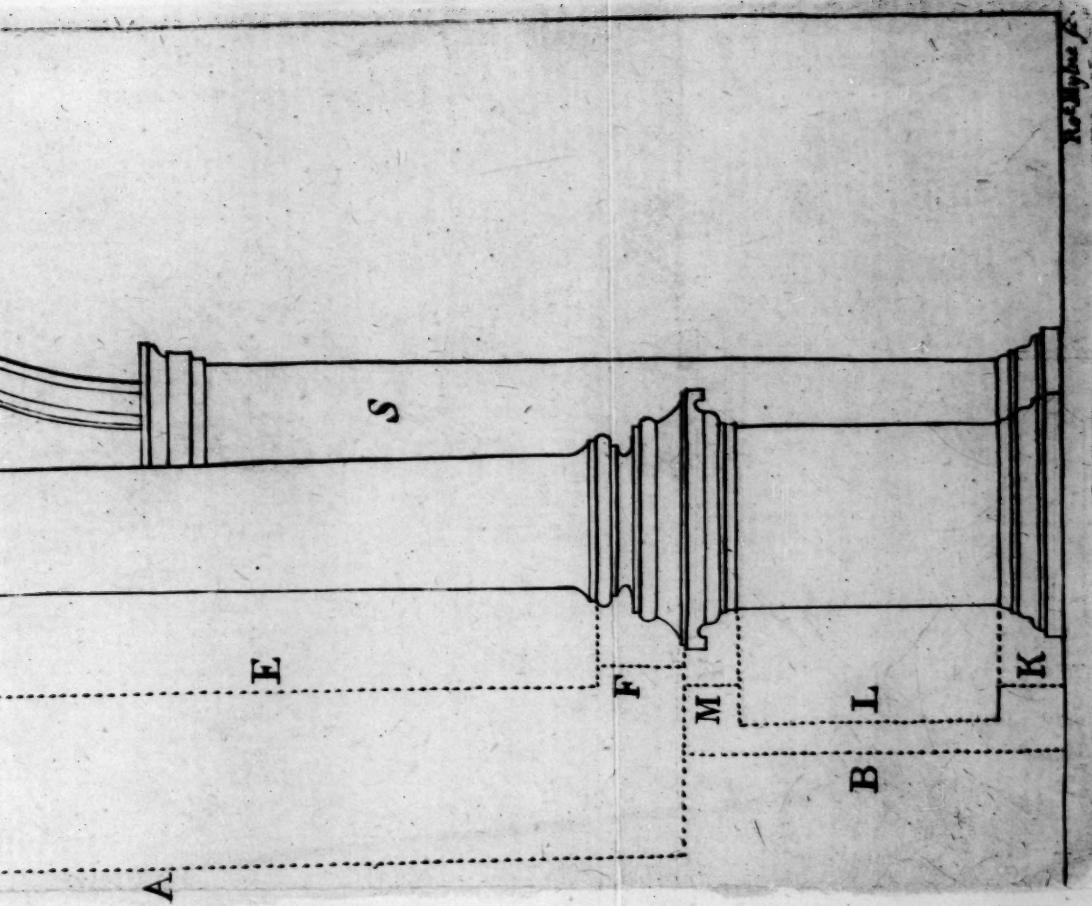
On its Altitude, as a Diameter, describe a Semicircle.

PROB.

Plate 14.







## PROB. II.

Plate XV,  
FIG. 3.*TO describe an Echin.*

Divide  $BD$  into Three equal Parts at  $a$  and  $b$ , set off  $BC = Bb$ , make  $BE = Ba + \frac{1}{4}ab$ . So  $E$  is the Center, and  $EC$  the Radius.

For  $DE_q = DB_q + BE_q = \frac{12}{12}BD^2 + \frac{5}{12}BD^2 = \frac{144}{144}BD_q + \frac{25}{144}BD_q = \frac{169}{144}BD_q$ , therefore  $DE = \frac{13}{12}BD = \frac{8}{12}BD + \frac{5}{12}BD = BC + BE = EC$ . Q. E. D.

Or thus.

Make  $BC = \frac{2}{3}BD$ , on the Centers  $c$  and  $D$ , with the Distance  $CD$ , strike Two Arches, their Interfection is the Center of  $CD$ .

## PROB. III.

Plate XV,  
FIG. 4.*TO describe a Dorick Cymaise.*

Bisect  $AB$  the Altitude in  $d$ , make the Projection  $AE = Ad$ , bisect  $Ad$  in  $c$ , make  $BF = AB + AC$ . So  $F$  is the Center.

Draw  $EG \parallel AB$ . So  $BG = AE = \frac{2}{4}AB$ , and  $GF = \frac{3}{4}AB$ , then  $EF_q = EG_q + GF_q = \frac{16}{16}AB_q + \frac{9}{16}AB_q = \frac{25}{16}AB_q$ , there-

206 *A short Introduction*

therefore  $EF = \frac{5}{4} AB = BF$ . Q.  
E. D.

Or thus.

Make the Projection  $AE = \frac{1}{2} AB$ , and on E, B, as Centers, describe Two Arches, their Intersection shall be the Center sought.

Plate XV.  
FIG. 5.

PROB. IV.

*TO describe a Trochile.*

Divide the Altitude  $AB$  into Three equal Parts at  $D$  and  $E$ , make  $AC = AD$ . On  $AD$  and  $DB$  raise the Squares  $AF$  and  $BG$ ; it is plain,  $F$  is the Center of the Arch  $CD$ , and  $G$  that of  $DK$ .

Plate XV.  
FIG. 6.

PROB. V.

*TO describe a Lesbian Cymaïse.*

Bisect the Altitude  $ON$  at  $a$ , make the Projection  $OL = oa$ , and  $OI = \frac{1}{4} OA$ , and  $NK = IL$ : I say,  $I$  is the Center of  $LM$ , and  $K$  the Center of  $MN$ .

Draw  $MG$  &  $INK$ . So  $NG = aM = \frac{1}{2} OL = \frac{1}{4} ON = \frac{2}{8} ON$ , and  $NK = \frac{5}{8} ON$ , there-



therefore  $GK = NK - NG = \frac{3}{8}ON$ .  
 Now  $MK_q = MG_q + GK_q = \frac{16}{64}ON_q + \frac{9}{64}ON_q = \frac{25}{64}ON_q$ , whence  $MK = \frac{5}{8}ON = NK$ . *Q. E. D.*

After the same Manner we may demonstrate *I* to be the Center of the Arch *LM*.

Or thus.

Make  $OL = \frac{1}{2}ON$ , draw *LN*, and bisect it at *M*, on the Centers *L*, *M*, with the Distance *LM*, strike Arches whose Intersection is the Center of *LM*, do the same at *M* and *N*.

### PROB. VI.

Plate XV.  
FIG. 7.

*TO describe a Sima.*

Make  $AC = AB$ , bisect *AC* in *D*, on *AD*, *DC* raise the Squares *AF*, *DG*; it is plain, that *G* is the Center of *CF*, and *E* the Center of *FB*.

### PROB. VII.

Plate XV.  
FIG. 8.

*TO describe a Chanfrain.*

Bisect *AB* in *D*, make  $BC = AD$ , draw *AE*  $\perp$  *AB*, and  $AE = \frac{5}{4}AB$ . So *E* is the Center, and *EA* the Radius. The De-

Demonstration is the same as in PROB. III.

N. B. Plain Members, according to their different Magnitudes and Situation, take different Names. So a Reglet on the Top of a Cornice, of the Capital of the Architrave, and of the Frize, is called *A Supercilie*; and in the same Manner greater Members take different Denominations from their different Situation: So the rectangular Member in the Base of the Pedestal, is called *The Socle*, in the Base of the Column it is called *The Plinthe*, in the Cornices, *The Crown* or *Corona*, in the Capital *Abaco*, *Abaque*, or *Gorgerin*, in the Architrave, *Fascia*, *Tania*, or *Orle*.

N. B. 2. Because the Eye is delighted with Variety, it is manifest that the same Members cannot be immediately conjoyned, and therefore betwixt curvilineal Members Reglets must be interposed, and betwixt plain Members, Astragals.

*Def. Essential Members* are such as are necessarily in the same Part of every Order. So the *De* or *Dy* is essential in the Pedestal, the *Orle* and *Chanfrain* in the Column, in the Base of the Pillar the *Plinthe*, in the Capital the *Gorgerin*, in the Architrave the *Fascia* or *Bande*, and in the Cornice the *Doucine* and *Supercilie*.

N. B. 3. The Two *Cymaifes*, *Ove* and *Gueule*, having their Projections increasing, are fit Members of the Cornices and Capital, because these Parts likewise increase, but the *Tore* and *Scotie* cannot enter these Parts.

N. B. 4. All Members, but the *Ove*, are convenient for the Bases of the Pedestal and Pillar. For the Two *Cymaifes* and *Gueule* inverted, contribute to the decreasing of these Parts upwards, as well as the *Scotie*; the *Tore* does not prevent it, but it, being convex, makes the inverted *Ove* of no Use in these Parts.

N. B. 5. Besides the *Mouluures* already described, the ancient *Greeks* and *Romans* used other Ornaments, as in the  
Capit

Plate 15.

fig: 1<sup>re</sup>.

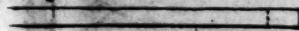


fig: 2.

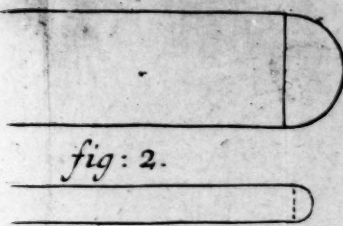


fig: 3.

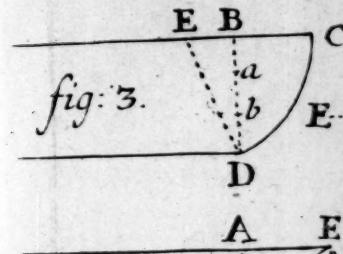


fig: 4.

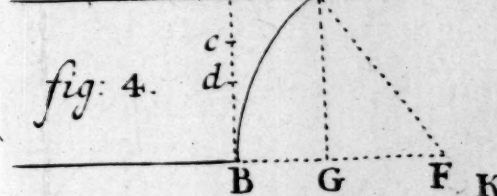


fig: 5.

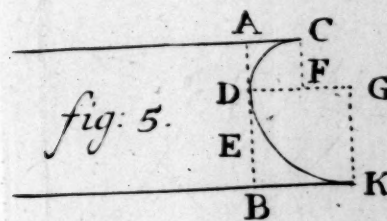


fig: 6.

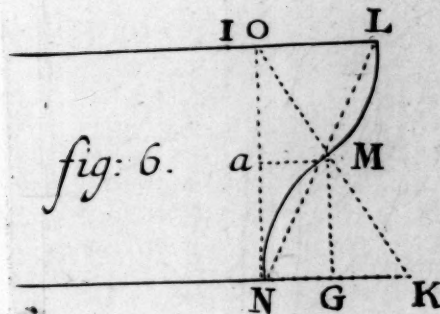


fig: 7.

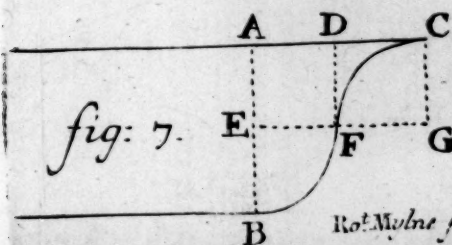
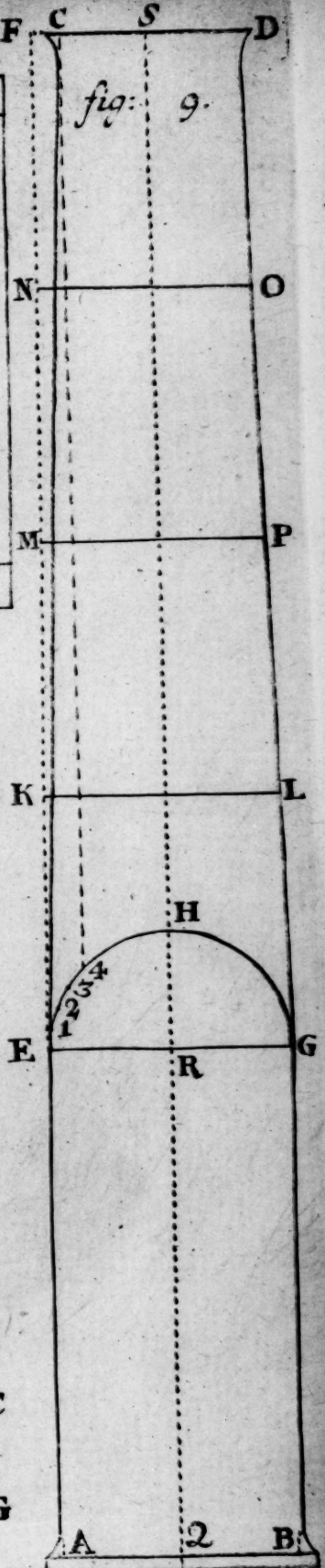


fig: 8.



fig: 9.



Rot. Molne sc.



Capitals, Volute, and the Leaves and Stalks of Acanthus or Brankursin; in the Architrave, Triglyphes *p*, with their Gouttes *r*, and in the Cornices, Mutules or Modillions *n*, and Denticules.

Æ B. 6. The Distance betwixt Two Triglyphes, Mutules or Denticules, is called *A Metope*.

## PROB. VIII.

Plate XVI.  
FIG. 1.

**TO** describe a Volute or Scroll.

Divide *AB* into Eight equal Parts,

\* on *oq* the Fifth Part from *A*, as a Diameter, describe a Circle for the Eye of the Volute, draw the perpendicular Diameter *ns*, and continue it, \* Suppose *oq*, in Fig. 2. to be the Fifth Part.

in the Circle inscribe the Square *onqs*, bisect its Sides with the Lines *ac*, *nb*, divide each of these Lines into Six equal Parts, so you have Twelve Centers, ●, 1, , x, ||, \*, 8, 9, *a*, *b*, ‡‡, *c*. On the Center ●, with the Distance ●*o*, describe a Quadrant, on the Center 1 describe a Quadrant, &c. as the compounding Part of the Volute.

Or thus.

On † *cd*, the Fifth Part from *B*, describe a Circle for the Eye of the Volute, bisect *od*, *oc*, at 8 and 11, divide 8 11, into Six equal Parts, at 7, 3, *o*, ●, 4, on 8 11, raise the

*D d*

Square

Plate XVI.  
FIG. 3.

† Suppose *cd*, in Fig. 4. to be the Fifth Part.

## 210 *A short Introduction*

Square 119, draw  $o9, o10$ . At the several Divisions of 811, raise Perpendiculars to  $cd$ , these shall divide  $o9, o10$ , into Three equal Parts each, so you have Twelve Centers, ●, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11. On these Centers describe the Quadrants, &c.

*N. B.* The First of these is *Palladio's*, the other *V. truvius's* Volute.

*Obs.* As, in the Infancy of Architecture, Pillars came in Place of Trees, which the ancient *Greeks* made Use of in their Buildings, so these Pillars must taper as Trees do.

### PROB. IX.

*TO diminish any Pillar.*

Having determined  $qs$  the Altitude, and  $cd$  the Diameter at the Top, for Example, make  $cd = \frac{2}{3} AB$ , the Diameter at the Base, divide the whole Altitude  $qs$  or  $AE$ , into Three equal Parts, of which  $AE$  is one, for that Distance  $AE$  let the Diameter be every where the same, divide  $EF$  into any Number of equal Parts at  $k, m, n$ , and draw  $EG, KL$ , &c. ||  $AB$ , on the Dia-

Diameter  $EG$  describe the Semicircle  $EHG$ , draw  $C4 \parallel AE$  continued, divide  $E4$  into as many equal Parts as  $EF$  is divided into, through the several Divisions of  $E4$  draw Lines  $\parallel EF$ , mark the Points where these Lines meet with  $KL$ ,  $MP$ , and joyn these Points with right Lines, and it is done.

## DEFINITIONS.

Plate XVII

**T**HE *Tuscan Order* is altogether simple and plain, its Capital has no Volutes, and its Frize is void of all Ornaments.

The *Dorick Order* has no Volutes in its Capital, but is distinguished by Triglyphes in its Frize.

The *Ionick Order* has Four Volutes in its Capital, but has no Leaves.

The *Roman* or *Composite Order* has Eight Volutes, and Two Series of Leaves.

The *Corinthian Order* has Three Series of Leaves and Stalks, and Sixteen Volutes.

*Module* is the Semidiameter of the lower Part of the Fust of the Pillar.

*Minute* is the Thirtieth Part of a Module.



*Ecphora* is the Distance of the Extremity of any Part or Member from the Axis of the Pillar continued.

Here we shall lay down the Altitudes and *Ecphoræ* of the Members of the several Orders according to *Palladio* and *Scamozzi*, taking the Semidiameter of the lower Part of the Fust for our Module or common Measure.

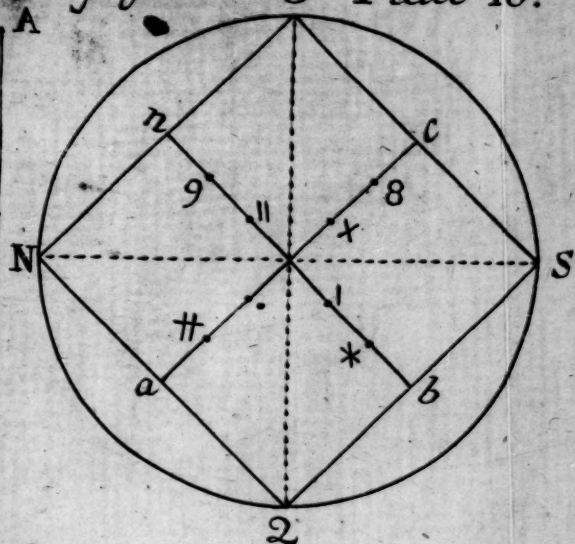
*The TUSCAN ORDER according to  
PALLADIO.*

*Palladio*, in this Order, makes the Socle altogether plain and high, sometimes 1 Module, sometimes 2. its Breadth he makes  $2^m \frac{3}{4}$ . or  $2^m. 22' \frac{1}{2}$ . and consequently its *Ecphora* is  $1^m. 11' \frac{1}{4}$ .

He gives  $14^m$  to the whole Column, including its Base and Capital, and of that  $\frac{1}{4}$ , or  $3^m. 15'$ . to the Entablement or Travaifon, so that the whole Order, taking in the Socle of  $2^m$ . is  $19^m. 15'$ . or making the Socle  $1^m$  only, the Whole is  $18^m. 15'$ . and is divided into its several Parts, and they into their Members or Moulures, thus.

Parts.





Volute

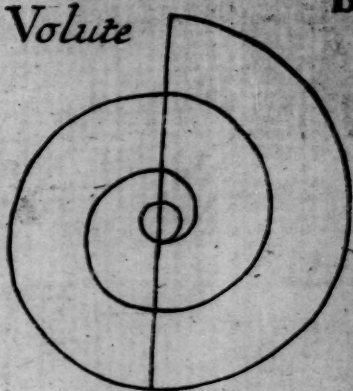


fig: 3.

fig: 1.

fig: 4.

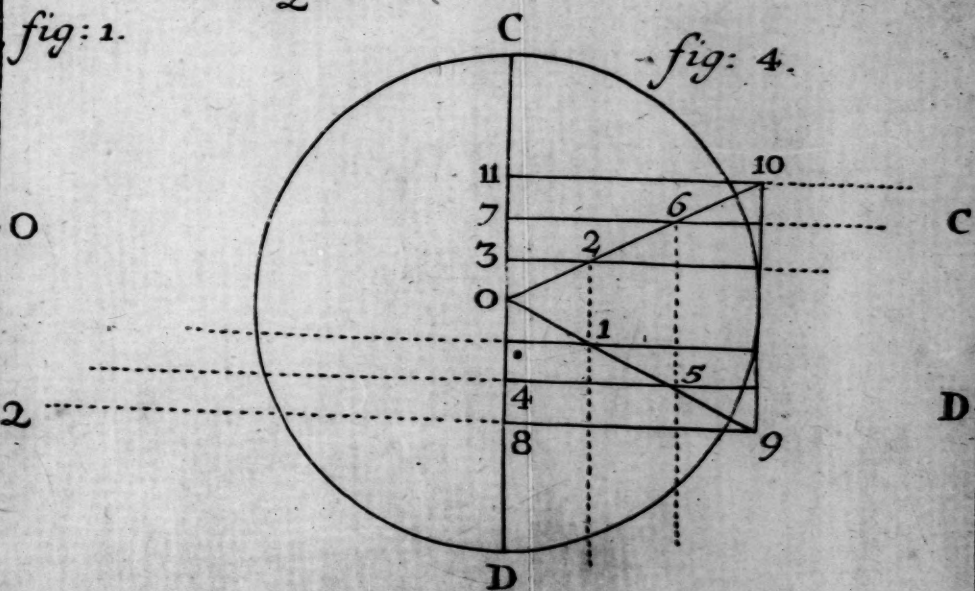
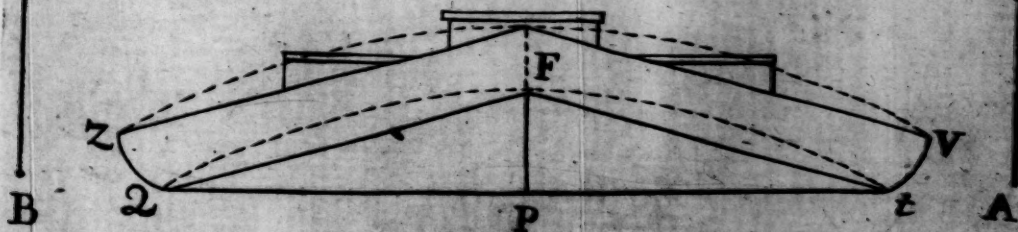


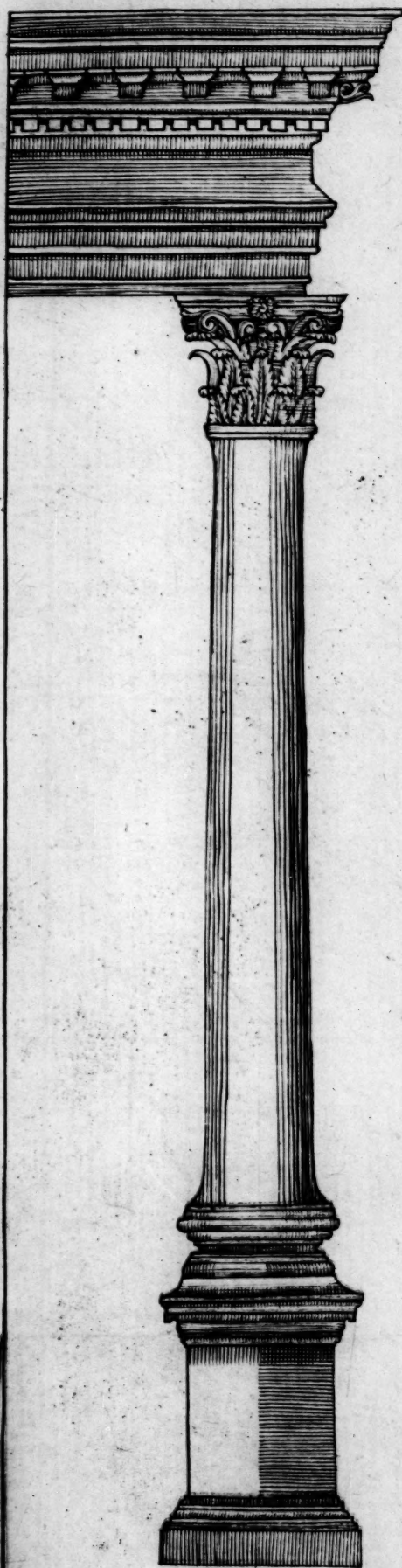
fig: 5.



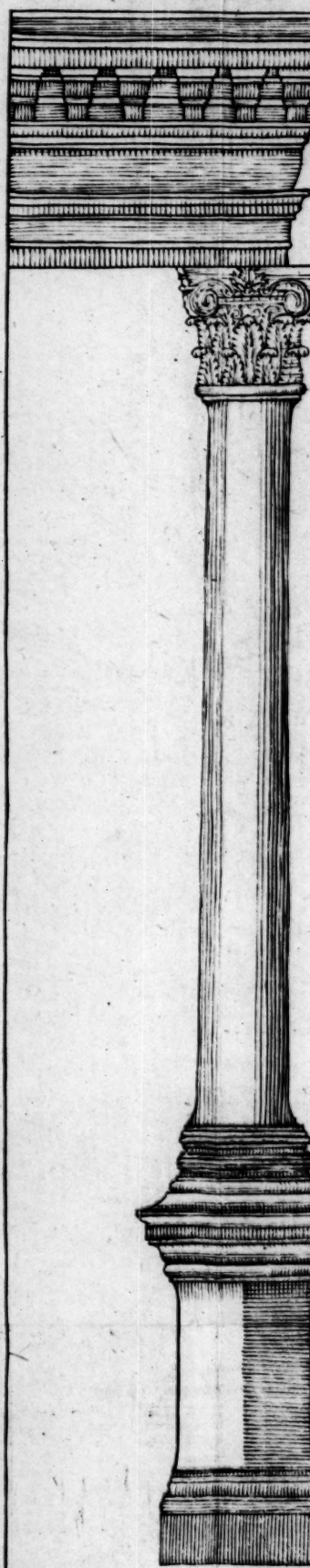
Robt. Hylton sc.



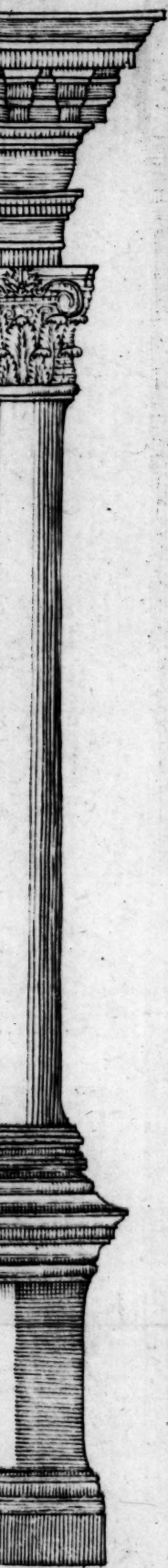
CORINTHIAN



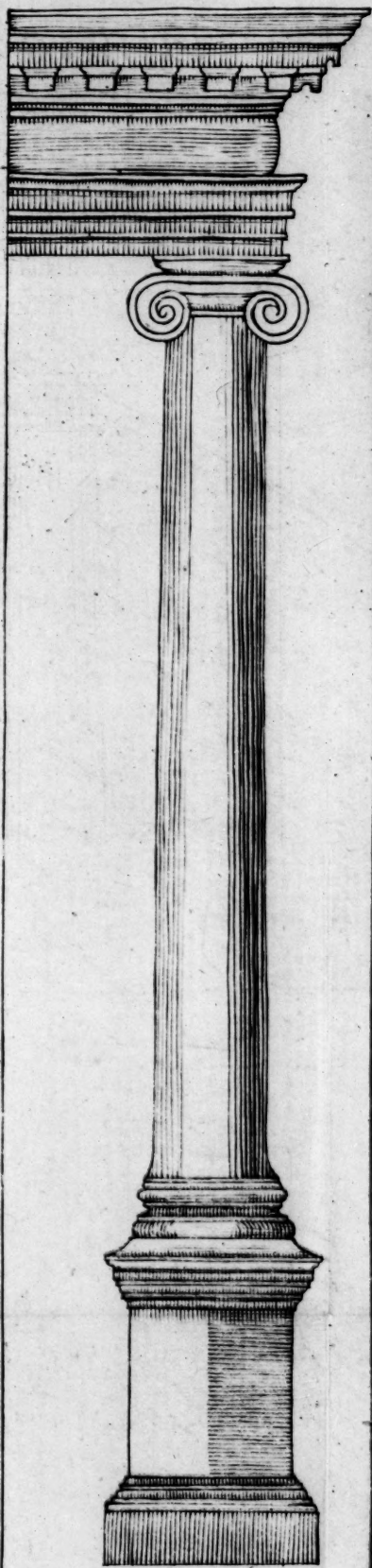
COMPOSIT



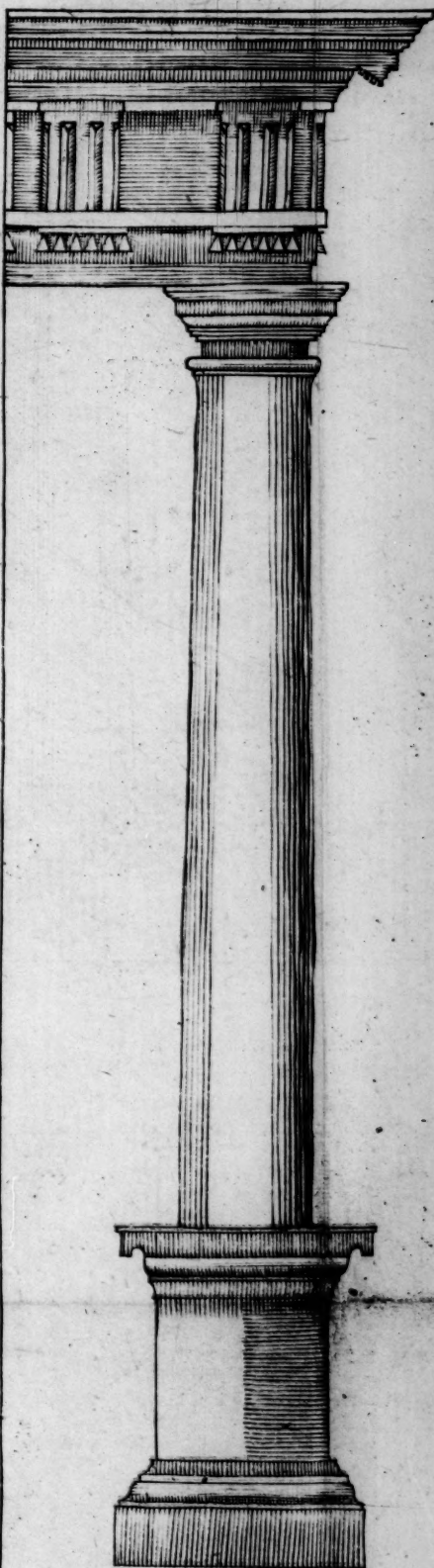
SITE



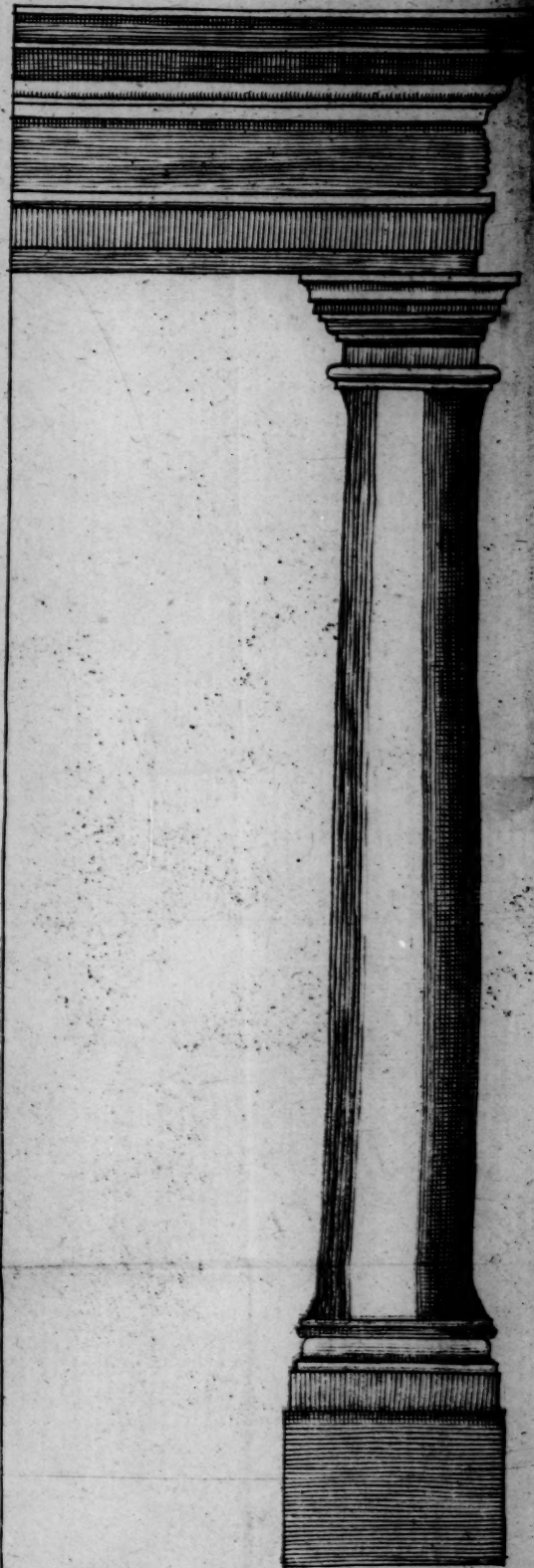
IONIC



DORIC



TUSCAN ORDER





| Parts.                            | Members.               | Heights. |                  | Ecphora.      |                  |
|-----------------------------------|------------------------|----------|------------------|---------------|------------------|
|                                   |                        | m        |                  | m             |                  |
| COLUMN 14 <sup>m</sup> .          | Socle,                 | 1        | —                | 1             | 11 $\frac{1}{4}$ |
|                                   | Plinthe,               | —        | 15               | —             | 10               |
|                                   | Reglet,                | —        | 1 $\frac{1}{2}$  | 1             | 7 $\frac{3}{4}$  |
|                                   | Gueule,                | —        | 9 $\frac{1}{2}$  | 1             | 3                |
|                                   | Astragal,              | —        | 4                | 1             | 5                |
|                                   | Orle infer.            | —        | 2                | 1             | 3                |
|                                   | Fust simply,           | 11       | 22 $\frac{1}{2}$ | <i>Supra</i>  | 25               |
|                                   | Orle super.            | —        | 1 $\frac{1}{2}$  | —             | 28               |
|                                   | Astragal,              | —        | 4                | 1             | —                |
|                                   | Gorgerin,              | —        | 8 $\frac{1}{2}$  | —             | 25               |
|                                   | Orle,                  | —        | 1 $\frac{1}{2}$  | —             | 27               |
|                                   | Cymaïse inv.           | —        | 8 $\frac{1}{2}$  | —             | —                |
|                                   | Reglet,                | —        | 1                | —             | 29 $\frac{1}{2}$ |
|                                   | Goutiere,              | —        | 5 $\frac{1}{2}$  | 1             | 1                |
|                                   | Cavett,                | —        | 3                | 1             | 2 $\frac{1}{2}$  |
|                                   | Reglet 2.              | —        | 2                | 1             | 5                |
| ENTABLEMENT 3 <sup>m</sup> . 15'. | Bande 1.               | —        | 12 $\frac{1}{2}$ | —             | —                |
|                                   | Bande 2.               | —        | 17 $\frac{1}{2}$ | —             | 27               |
|                                   | Reglet with Chanfrain, | —        | 5                | 1             | —                |
|                                   | Frize,                 | —        | 26               | —             | 25               |
|                                   | Cavett,                | —        | 8                | —             | —                |
|                                   | Filet,                 | —        | 1 $\frac{1}{2}$  | 1             | 2 $\frac{1}{2}$  |
|                                   | Ove,                   | —        | 9                | 1 <i>Sup.</i> | 11 $\frac{1}{2}$ |
|                                   | * Goutiere,            | —        | 10               | 1             | 25               |
|                                   | Reglet,                | —        | 2                | 1             | 26               |
|                                   | Doucine,               | —        | 10               | —             | —                |
|                                   | Reglet.                | —        | 3 $\frac{1}{2}$  | 2             | 9                |

Bounded  
with a  
Curve of  
rustick  
Work.



## 214 *A short Introduction*

\* At  $4\frac{1}{2}$  from the Extremity of the Goutiere, there is a Scotie which joyns the Extremity of the Ove.

### *The TUSCAN ORDER according to SCAMOZZI.*

**I**N this Order *Scamozzi* gives to the Pillar, with its Base and Capital,  $15^m$ . and of that  $\frac{1}{4}$ . or  $3^m. 22\frac{1}{2}$ . to the Pedestal, and as much to the Entablement or Travaison; so that the whole Order contains  $22^m \frac{1}{2}$ .

The Heights and Ecphoræ of the several Parts, and their Members, we shall shew in Modules and Minutes, after the same Manner as we did in *Palladio's Tuscan Order*.

| Parts.                    |                       | Members.   | Heights.          | Ecphoræ. |
|---------------------------|-----------------------|------------|-------------------|----------|
|                           |                       |            | m                 | m        |
| PED. $3^m. 22\frac{1}{2}$ | 1 <sup>m</sup> .      | Socle,     | 1 —               | 1 13     |
|                           | 2 <sup>m</sup> .      | Dy or De,  | 2 —               | 1 10     |
|                           | Cornice.              | Chanfrain, | — $5\frac{1}{2}$  | 1 11     |
|                           |                       | Filet,     | — 1               | 1 14     |
|                           |                       | Goutiere,  | — $12\frac{1}{2}$ | 1 15     |
|                           |                       | Reglet,    | — $3\frac{1}{2}$  | 1 16     |
| CO-                       | Base 1 <sup>m</sup> . | Plinthe,   | — 18              | 1 10     |
|                           |                       | Tore,      | — 12              | 1 10     |

Parts

| Parts:      |                                            | Members.    | Heights. |                  | Ecphoræ. |                  |
|-------------|--------------------------------------------|-------------|----------|------------------|----------|------------------|
|             |                                            |             | m        | '                | m        | '                |
| LUMN        | Fust 13 <sup>m</sup> .                     | Reglet with | —        | 3                | I        | 2 $\frac{1}{8}$  |
|             |                                            | Chanfrain,  | 12       | 22 $\frac{1}{2}$ | Sup. 22  | $\frac{1}{2}$    |
|             |                                            | Trunk 5.    | —        | 1 $\frac{1}{2}$  | —        | 25 $\frac{1}{2}$ |
|             |                                            | Filet with  | —        | 3                | —        | 27               |
|             | Capital 1 <sup>m</sup> .                   | Chanfrain,  | —        | 8 $\frac{3}{4}$  | —        | 22 $\frac{1}{2}$ |
|             |                                            | Astragal,   | —        | 2 $\frac{1}{2}$  | —        | 24               |
|             |                                            | Ove,        | —        | 7 $\frac{1}{2}$  | —        | 25 $\frac{1}{2}$ |
|             |                                            | Goutiere,   | —        | 7 $\frac{1}{2}$  | I        | 29               |
|             |                                            | Reglet,     | —        | 3                | I        | I                |
|             | Archit. 1 <sup>m</sup> . 2 $\frac{1}{2}$ . | Bande 1.    | —        | 11               | —        | 22 $\frac{1}{2}$ |
|             |                                            | Bande 2.    | —        | 16 $\frac{1}{2}$ | —        | 23 $\frac{1}{2}$ |
|             |                                            | Filet,      | —        | 1 $\frac{1}{3}$  | —        | 25               |
|             |                                            | Reglet,     | —        | 3 $\frac{2}{3}$  | —        | 26               |
| ENTABLEMENT | Frieze 1 <sup>m</sup> . 13.                | Plat Bande  | I        | 11               | —        | 22 $\frac{1}{2}$ |
|             |                                            | Reglet,     | —        | 2                | —        | 24               |
|             | Cornice 1 <sup>m</sup> . 9 $\frac{1}{2}$ . | Cavett,     | —        | 5 $\frac{1}{2}$  | —        | —                |
|             |                                            | Reglet,     | —        | 1 $\frac{1}{2}$  | —        | 28 $\frac{1}{2}$ |
|             |                                            | Ove,        | —        | 6                | I        | 1 $\frac{1}{2}$  |
|             |                                            | Cavett 2.   | —        | 1 $\frac{3}{4}$  | I        | 5                |
|             |                                            | Filet,      | —        | 1 $\frac{1}{2}$  | I        | 19 $\frac{1}{2}$ |
|             |                                            | Goutiere,   | —        | 9                | I        | 20 $\frac{1}{2}$ |
|             |                                            | Reglet,     | —        | 1 $\frac{1}{2}$  | I        | 22 $\frac{1}{2}$ |
|             |                                            | Doucine,    | —        | 8                | —        | —                |
|             |                                            | Filet,      | —        | 1 $\frac{1}{2}$  | 2        | 2 $\frac{1}{2}$  |
|             |                                            | Reglet.     | —        | 2 $\frac{3}{4}$  | 2        | 1 $\frac{1}{2}$  |

On the  
Plat bande  
and Reglet  
are plain  
Modillions  
in Form  
of Tri-  
glyphes.

The

*The DORICK ORDER according to  
PALLADIO.*

*Palladio* gives to the simple Fust of this Column (*i. e.* without Base or Capital)  $14^m$ . the De of the Pedestal he makes  $2^m$ .  $20'$ . every Way; the Altitude of its Base he makes  $\frac{1}{2}$  the Dy, or  $1^m$ .  $10'$ . and that of its Cornice  $\frac{1}{4}$  the Dy, or  $20'$ . so the Height of the whole Pedestal is  $4^m$ .  $20'$ . and its Parts, *viz.* the Cornice, Base and Dy, are as 1, 2, 4.

He sometimes places the Fust of the Column on the Pedestal, without any Base, sometimes he allows it an *Attick* Base of  $1^m$ . high, and a Capital  $1^m$ . high.

The whole Entablement is  $\frac{1}{4}$  the Column, and its Capital taken together, *i. e.*  $3^m$ .  $22'\frac{1}{2}$ . of which the Architrave has  $1^m$ . the Frize  $1^m$ .  $15'$ . and the Cornice  $1^m$ .  $7'\frac{1}{2}$ . So the whole Order, without a Base, is in Height  $23^m$ .  $12'\frac{1}{2}$ . and with a Base  $24^m$ .  $12'\frac{1}{2}$ .

He has Two Dispositions of the Parts and Members of his Pedestal.



## DISPOSITION I.

| Parts.                         |                            | Members.   | Heights.        |                  | Ecphoræ.         |                  |
|--------------------------------|----------------------------|------------|-----------------|------------------|------------------|------------------|
|                                |                            |            | m               |                  | m                |                  |
| PEDESTAL 4 <sup>m</sup> . 20'. | Base 1 <sup>m</sup> . 10'. | Socle,     | —               | 27 $\frac{1}{2}$ | I                | 20 $\frac{1}{4}$ |
|                                |                            | Tore,      | —               | 5                | —                | —                |
|                                |                            | Reglet,    | —               | 1 $\frac{1}{4}$  | I                | 16 $\frac{1}{4}$ |
|                                |                            | Reglet 2.  | —               | 1 $\frac{1}{4}$  | I                | 15 $\frac{1}{4}$ |
|                                |                            | Chanfrain, | —               | 5                | —                | —                |
|                                | Cornice 20'.               | Dy or De,  | 2               | 20               | I                | 10               |
|                                |                            | Chanfrain, | —               | 5                | I                | 11               |
|                                |                            | Reglet,    | —               | 1 $\frac{1}{4}$  | I                | 16               |
|                                |                            | Reglet 2.  | —               | 1 $\frac{1}{4}$  | I                | 17               |
|                                |                            | Doucine,   | —               | 9                | —                | —                |
|                                | Reglet,                    | —          | 3 $\frac{1}{2}$ | I                | 26 $\frac{1}{4}$ |                  |

## DISPOSITION II.

|                               |                           |           |                     |                    |
|-------------------------------|---------------------------|-----------|---------------------|--------------------|
| PEDESTAL 4 <sup>m</sup> . 20. | Base 1 <sup>m</sup> . 10. | Socle,    | — 26                | I 26 $\frac{1}{4}$ |
|                               |                           | Reglet,   | — 1 $\frac{1}{3}$   | I 25 $\frac{1}{4}$ |
|                               |                           | Doucine,  | — 7 $\frac{1}{3}$   | — —                |
|                               |                           | Reglet,   | — 1 $\frac{1}{3}$   | I 15 $\frac{1}{4}$ |
|                               |                           | Cavett,   | — 4                 | — —                |
|                               | Cornice 20.               | Dy,       | 2 20                | I 10               |
|                               |                           | Cavett,   | — 4                 | — —                |
|                               |                           | Reglet,   | — 1                 | I 15 $\frac{1}{2}$ |
|                               |                           | Ove,      | — 6 $\frac{1}{2}$   | I 18 $\frac{3}{4}$ |
|                               |                           | Goutiere, | — † 6 $\frac{1}{2}$ | I 25 $\frac{1}{2}$ |
|                               |                           | Reglet,   | — 2                 | I 26 $\frac{1}{2}$ |

E e

Parts.

\* To be  
joyn'd with  
an Arch of  
a Circle to  
the Reglet  
below.

| Parts.                                          | Members.                     | Heights.                             |                     | Ecphora. |                  |
|-------------------------------------------------|------------------------------|--------------------------------------|---------------------|----------|------------------|
|                                                 |                              | m                                    |                     | m        |                  |
| C<br>O<br>L<br>U<br>M<br>N<br>16 <sup>m</sup> . | Attick Base 1 <sup>m</sup> . | Plinthe, *                           | — 10                | 1        | 10               |
|                                                 |                              | Tore lower,                          | — 7 $\frac{1}{2}$   | —        | —                |
|                                                 |                              | Filet,                               | — 1 $\frac{1}{4}$   | 1        | 6 $\frac{1}{2}$  |
|                                                 |                              | Scotie,                              | — 5 $\frac{3}{4}$   | —        | —                |
|                                                 |                              | Filet 2.                             | — 1 $\frac{1}{4}$   | 1        | 5                |
|                                                 |                              | Tore above,                          | — 4 $\frac{1}{4}$   | 1        | 6 $\frac{1}{2}$  |
|                                                 | Trunk 14 <sup>m</sup> .      | Anneau or<br>Ring with<br>Chanfrain, | — 1 $\frac{1}{4}$   | 1        | 3 $\frac{1}{2}$  |
|                                                 |                              | Trunk,                               | 13 23 $\frac{3}{4}$ | above 25 | —                |
|                                                 |                              | Anneau a-<br>bove,                   | — 1 $\frac{1}{2}$   | —        | 26 $\frac{3}{4}$ |
|                                                 |                              | Astragal,                            | — 3 $\frac{1}{2}$   | 1        | —                |
|                                                 | Capital 1 <sup>m</sup> .     | Gorgerin,                            | — 9                 | —        | 25               |
|                                                 |                              | Anneau 1.                            | — 3 $\frac{1}{3}$   | —        | 26               |
|                                                 |                              | Anneau 2.                            | — —                 | —        | 27               |
|                                                 |                              | Anneau 3.                            | — —                 | —        | 28               |
|                                                 |                              | Ove,                                 | — 6 $\frac{1}{2}$   | —        | —                |
|                                                 |                              | Goutiere,                            | — 6 $\frac{3}{4}$   | 1        | 3                |
|                                                 |                              | Talon,                               | — 2 $\frac{2}{3}$   | —        | —                |
|                                                 |                              | Filet,                               | — 1 $\frac{3}{4}$   | 1        | 7                |
|                                                 | Architrave                   | Bande,                               | — 11                | —        | 25               |
|                                                 |                              | Bande 2.                             | — 9 $\frac{1}{3}$   | —        | 26               |
|                                                 |                              | Bandelette<br>of Gouttes,            | — 3 $\frac{2}{3}$   | —        | 28               |
|                                                 |                              | Filet on the<br>Gouttes,             | — 1 $\frac{1}{2}$   | —        | —                |

Parts.

| Parts.                                                      | Members.                                                            | Heights. |                               | Ecphoræ.            |                                |
|-------------------------------------------------------------|---------------------------------------------------------------------|----------|-------------------------------|---------------------|--------------------------------|
|                                                             |                                                                     | m        |                               | m                   |                                |
| TABLEMENT 3 <sup>m</sup> . 22 <sup>1</sup> / <sub>2</sub> . | 1 <sup>m</sup> .<br>Reglet Capital of Gouttes.                      | —        | 4 <sup>1</sup> / <sub>2</sub> | —                   | 28 <sup>1</sup> / <sub>4</sub> |
|                                                             | 1 <sup>m</sup> . 15'.<br>Triglyphes, Void above Triglyphes, Metope, | 1 ‡ 10   | — 5                           | 1 broad. Metope. 10 | Projection of Metopes 3'.      |
|                                                             |                                                                     | 1        | 15                            | 1                   | 15                             |
|                                                             | Bandelette where are the Capitals of the Triglyphes,                | —        | 5                             | —                   | 29 <sup>1</sup> / <sub>2</sub> |
|                                                             | Cavett,                                                             | —        | 5                             | —                   | —                              |
|                                                             | Filet,                                                              | —        | 1                             | 1                   | 4                              |
|                                                             | Ove,                                                                | —        | 6                             | 1 *                 | 8 <sup>1</sup> / <sub>2</sub>  |
|                                                             | Goutiere,                                                           | —        | 8                             | 2 *                 | 3 <sup>1</sup> / <sub>2</sub>  |
|                                                             | Talon,                                                              | —        | 3 <sup>1</sup> / <sub>4</sub> | —                   | —                              |
|                                                             | Filet 2.                                                            | —        | 3 <sup>3</sup> / <sub>4</sub> | 2                   | 7 <sup>1</sup> / <sub>2</sub>  |
| Cornice 1 <sup>m</sup> . 7 <sup>1</sup> / <sub>2</sub> .    | Gueule,                                                             | —        | 6 <sup>1</sup> / <sub>2</sub> | —                   | —                              |
|                                                             | Reglet,                                                             | —        | 2                             | 2                   | 15                             |

† In the Goutiere of the Cornice of the Pedestal below, there is a Scotie that joyns the Ove, and leaves on the exterior Part of the Goutiere 2<sup>1</sup>/<sub>2</sub>.

‡ The Middle of the Triglyphe is directly above that of the Pillar.



## 220 *A short Introduction*

On each Side of the Triglyphe make a Canal, in Breadth  $2\frac{1}{2}$ . then Two Canals within of 5' each, and allow 5' of Interstice betwixt Two Canals, the Demimetope is 10'. their Projection 3'. directly below the Triglyphes are always the Gouttes.

\* The Difference of the *Ecphoræ* of the Goutiere and Ove is 25'. to be disposed thus.

|                                 |           |    |
|---------------------------------|-----------|----|
| Betwixt the Ove and Goutte      | - - -     | 2  |
| For Three Gouttes               | - - - -   | 15 |
| For a Reglet                    | - - - - - | 2  |
| For Two Reglets placed as Steps | -         | 2  |
| For a Scotie                    | - - - - - | 2  |
| For a Reglet                    | - - - - - | 2  |

In this and the following Orders there are often Canals in the Fust of the Column ; these cannot exceed Twenty in Number. To make them, inscribe in the lower Base of the Fust a Polygon of Twenty Sides, admitting one of these Sides to move upwards, and diminish in the same Manner as the Pillar does, you have one Canal.

To

To hollow these Canals. On the Side of the Polygon describe a Square, the Point of Intersection of its Two Diagonals is the Center of the Hollow.

The Interstice betwixt Two Hollows is called *A Listel*.

The Hollow is sometimes continued to the whole Length of the Fust, sometimes to the Length of the Two upper Third Parts, and sometimes the Hollows of the first Third Part are filled up with Batons or Tores.

*The DORICK ORDER according to SCAMOZZI.*

*Scamozzi* gives to the Column, its Base and Capital  $17^m$ . and of that he allows  $\frac{4}{15}$ , or  $4^m. 15'$ . to the Pedestal, and  $\frac{1}{4}$ , or  $4^m. 7'\frac{1}{2}$ . to the Entablement. So the Height of the whole Order is  $25^m. 22'\frac{1}{2}$ .

Parts.

| Parts.   | Members.                    | Heights.                   |                     | Ecphoræ.      |                  |
|----------|-----------------------------|----------------------------|---------------------|---------------|------------------|
|          |                             | m                          | '                   | m             | '                |
| PEDESTAL | Base 1 <sup>m</sup> . 15'.  | Socle,                     | 1                   | 1             | 23 $\frac{1}{2}$ |
|          |                             | Tore,                      | — 4                 | —             | —                |
|          |                             | Reglet,                    | — 1                 | 1             | 20 $\frac{1}{2}$ |
|          |                             | Doucine,                   | — 6                 | —             | —                |
|          |                             | Reglet,                    | — 1                 | 1             | 14 $\frac{1}{2}$ |
|          |                             | Cavett,                    | — 3                 | —             | —                |
|          | Cornice 22 $\frac{1}{2}$ '. | Dy,                        | 2 7 $\frac{1}{2}$   | 1             | 11 $\frac{1}{4}$ |
|          |                             | Cavett,                    | — 4 $\frac{1}{2}$   | —             | —                |
|          |                             | Filet,                     | — 1 $\frac{1}{4}$   | 1             | 14               |
|          |                             | Ove,                       | — 5 $\frac{1}{2}$   | 1 <i>sup.</i> | 18               |
|          |                             | Goutiere,                  | — * 5 $\frac{1}{2}$ | 1 *           | 24               |
| COLUMN   | Base 1 <sup>m</sup> .       | Filet,                     | — 1 $\frac{1}{2}$   | 1             | 25 $\frac{1}{2}$ |
|          |                             | Reglet,                    | — 4                 | 1             | 27 $\frac{1}{2}$ |
|          |                             | Plinthe,                   | — 10 $\frac{1}{2}$  | 1             | 11 $\frac{1}{4}$ |
|          |                             | Tore below,                | — 8                 | —             | —                |
|          |                             | Orle,                      | — 1                 | 1             | 6 $\frac{3}{4}$  |
|          | Entabl 15 <sup>m</sup> .    | Scotie,                    | — 4                 | —             | —                |
|          |                             | Orle above,                | — 1                 | 1             | 4 $\frac{3}{4}$  |
|          |                             | Tore above,                | — 5 $\frac{1}{2}$   | 1             | 6 $\frac{3}{4}$  |
|          |                             | Orle below with Chanfrain, | — 2                 | 1             | 3 $\frac{1}{2}$  |
|          |                             | Trunk                      | 14 23               | above         | 24               |
|          |                             | Orle above with Chanfrain, | — 1 $\frac{1}{2}$   | —             | 26               |
|          |                             | Astragal,                  | — 3 $\frac{1}{2}$   | —             | 28               |



| Parts.      |                                 | Members.     | Heights. |                   | Ecphora. |                  |
|-------------|---------------------------------|--------------|----------|-------------------|----------|------------------|
|             |                                 |              | m        |                   | m        |                  |
| ENTABLEMENT | Capital 1 <sup>m</sup> .        | Gorgerin,    | —        | † 9 $\frac{1}{2}$ | —        | † 24             |
|             |                                 | Talon,       | —        | 2 $\frac{1}{2}$   | —        | —                |
|             |                                 | Anneau,      | —        | 1                 | —        | 27 $\frac{1}{2}$ |
|             |                                 | Ove,         | —        | 6 $\frac{1}{3}$   | —        | —                |
|             |                                 | Goutiere,    | —        | 6 $\frac{1}{3}$   | 1        | 2 $\frac{1}{2}$  |
|             |                                 | Talon,       | —        | 2 $\frac{2}{3}$   | —        | —                |
|             |                                 | Reglet,      | —        | 1 $\frac{1}{3}$   | 1        | 5 $\frac{1}{2}$  |
|             | Architrave 1 <sup>m</sup> . 5'. | Bande 1.     | —        | 12                | —        | 25               |
|             |                                 | Bande 2.     | —        | 11 $\frac{1}{2}$  | —        | 26               |
|             |                                 | Reglet of    | —        | 5                 | —        | —                |
|             |                                 | Gouttes,     | —        | —                 | —        | —                |
|             |                                 | Filet on     | —        | 1 $\frac{1}{2}$   | —        | —                |
|             |                                 | Gouttes,     | —        | —                 | —        | —                |
|             |                                 | Reglet,      | —        | 5                 | —        | 28 $\frac{1}{4}$ |
|             | Frieze 1 <sup>m</sup> . 14'.    | Triglyphe,   | 1        | 10                | 1 br.    | —                |
|             |                                 | Void above,  | —        | 5                 | —        | —                |
|             |                                 | Metope,      | 1        | 15                | 1 br.    | 15               |
|             |                                 | Demime-      | —        | —                 | — br.    | 9                |
|             |                                 | tope,        | —        | —                 | —        | —                |
|             |                                 | Projection,  | —        | —                 | —        | 3 $\frac{1}{2}$  |
|             | Cornice                         | Reglet       | —        | —                 | —        | —                |
|             |                                 | where are    | —        | 5                 | —        | 28 $\frac{1}{2}$ |
|             |                                 | the Capi-    | —        | —                 | —        | —                |
|             |                                 | tals of Tri- | —        | —                 | —        | —                |
|             |                                 | glyphes,     | —        | —                 | —        | —                |
|             |                                 | Talon,       | —        | 4 $\frac{1}{2}$   | —        | —                |
|             |                                 | Filet 1.     | —        | 1 $\frac{1}{4}$   | 1        | 3                |

Parts.

The Projection of the Goutiere has Three small Modillions, and a Fourth larger than the rest toward its exterior Part.

| Parts.                                           | Members.             | Heights.                        | Ecphora.                                     |
|--------------------------------------------------|----------------------|---------------------------------|----------------------------------------------|
|                                                  |                      | m                               | m                                            |
| 4 <sup>m</sup> . 7 <sup>1</sup> / <sub>2</sub> . | Bande of Denticules, | — 6                             | I 7                                          |
|                                                  | Filet 2.             | — 4 <sup>1</sup> / <sub>4</sub> | I 8                                          |
|                                                  | Ove,                 | — 5                             | I <i>sup.</i> 11 <sup>1</sup> / <sub>2</sub> |
|                                                  | Demicrueux,          | — 2 <sup>1</sup> / <sub>2</sub> | — —                                          |
|                                                  | Filet 3.             | — 1 <sup>1</sup> / <sub>4</sub> | I 13 <sup>1</sup> / <sub>2</sub>             |
|                                                  | Goutiere,            | — 7 <sup>1</sup> / <sub>2</sub> | 2 5                                          |
|                                                  | Talon,               | — 3 <sup>1</sup> / <sub>3</sub> | — —                                          |
|                                                  | Filet 4.             | — 1 <sup>1</sup> / <sub>4</sub> | 2 6 <sup>2</sup> / <sub>3</sub>              |
|                                                  | Doucine,             | — 6 <sup>2</sup> / <sub>3</sub> | — —                                          |
|                                                  | Reglet.              | — 2 <sup>1</sup> / <sub>2</sub> | 2 15                                         |

\* In the lower Part of the Goutiere of the Pedestal is a Scotie which joins the Ove.

† If the Column is canalled, Rosets and Leaves are carved on the Gorgon of the Capital, and Ovals on the Ove; and then instead of the Talon and Ring above the Gorgon, it is more ordinary to place a Reglet and Astragal.

The IONICK ORDER according to  
PALLADIO.

*Palladio*, in this Order, gives to the Column, with its Base and Capital,  $18^m$ . of that he allows  $\frac{7}{24}$ , or  $5^m$ .  $7\frac{1}{2}'$ . to the Pedestal, and  $\frac{1}{5}$ , or  $3^m$ .  $18'$ . to the Entablement. So the Height of the whole Ordonnance is  $26^m$ .  $25\frac{1}{2}'$ .

He has Two different Distributions of the Parts and Members of the Pedestal, and gives a Base either *Attick* or *Ionick* to the Column.

I. DISTRIBUTION *with Attick Base.*

| Parts.   | Members              | Heights. |                | Ecphoræ. |                 |
|----------|----------------------|----------|----------------|----------|-----------------|
|          |                      | m        | '              | m        | '               |
| PEDESTAL | Base $1^m$ . $12'$ . | —        | 28             | 1        | $26\frac{1}{4}$ |
|          |                      | —        | 4              | —        | —               |
|          |                      | —        | 1              | 1        | 24              |
|          |                      | —        | 5              | —        | —               |
|          |                      | —        | 1              | 1        | $17\frac{1}{4}$ |
|          |                      | —        | 3              | —        | —               |
|          | Dy,                  | 3        | 5              | 1        | $11\frac{1}{4}$ |
|          | Cornice              | —        | $3\frac{1}{2}$ | —        | —               |
|          |                      | —        | $1\frac{1}{2}$ | 1        | $14\frac{1}{4}$ |
|          |                      | —        | 4              | —        | —               |

F f

Parts.



# 226 *A short Introduction*

| Parts.      |                                         | Members. | Heights.    |                   | Ecphoræ.  |                  |
|-------------|-----------------------------------------|----------|-------------|-------------------|-----------|------------------|
|             |                                         |          | m           | '                 | m         | '                |
| C O L U M N | 5 <sup>m</sup> . 8'                     | 21'      | Filet       | — 1               | 1         | 21 $\frac{1}{4}$ |
|             |                                         |          | Goutiere,   | — 5               | 1         | 22 $\frac{1}{4}$ |
|             |                                         |          | Talon,      | — 3 $\frac{1}{2}$ | —         | —                |
|             |                                         |          | Reglet,     | — 2 $\frac{1}{2}$ | 1         | 26 $\frac{1}{4}$ |
|             | Attick Base 1 <sup>m</sup> .            |          | Plinthe,    | — 10              | 1         | 11 $\frac{1}{4}$ |
|             |                                         |          | Tore,       | — 7 $\frac{1}{2}$ | —         | —                |
|             |                                         |          | Orle,       | — 1 $\frac{1}{4}$ | 1         | 7 $\frac{1}{4}$  |
|             |                                         |          | Scotie,     | — 4 $\frac{3}{4}$ | —         | —                |
|             |                                         |          | Orle,       | — 1 $\frac{1}{4}$ | 1         | 4                |
|             |                                         |          | Tore,       | — 5 $\frac{1}{4}$ | 1         | 7 $\frac{1}{4}$  |
|             | Fust 16 <sup>m</sup> . 11 $\frac{2}{3}$ |          | Astragal,   | — 2 $\frac{1}{2}$ | 1         | 3 $\frac{1}{3}$  |
|             |                                         |          | Orle,       | — 1 $\frac{1}{3}$ | 1         | 2                |
|             |                                         |          | Trunk,      | 16 3              | above 26  | —                |
|             |                                         |          | Orle above, | — 1 $\frac{1}{2}$ | —         | 28 $\frac{1}{3}$ |
|             |                                         |          | Astragal,   | — 3 $\frac{1}{3}$ | Center 28 | 28 $\frac{1}{3}$ |

## II. DISTRIBUTION *with Ionick Base.*

|                 |                           |            |                   |   |                  |
|-----------------|---------------------------|------------|-------------------|---|------------------|
| P E D E S T A L | Base 1 <sup>m</sup> . 12' | Socle,     | — 28              | 1 | 26 $\frac{1}{4}$ |
|                 |                           | Filet,     | — 1               | 1 | 25 $\frac{1}{4}$ |
|                 |                           | Doucine,   | — 6               | — | —                |
|                 |                           | Tore,      | — 2 $\frac{1}{2}$ | 1 | 18 $\frac{1}{4}$ |
|                 |                           | Reglet,    | — 1               | 1 | 17               |
|                 |                           | Chanfrain, | — 3 $\frac{1}{2}$ | — | —                |
|                 | Cor-                      | Dy,        | 3 5               | 1 | 11 $\frac{1}{4}$ |
|                 |                           | Cavett,    | — 4               | — | —                |
|                 |                           | Filet,     | — 1               | 1 | 16 $\frac{1}{4}$ |
|                 |                           |            |                   |   |                  |

Parts.

| Parts.      |                                        | Members. | Heights.    |                    | Ecphoræ.      |                  |
|-------------|----------------------------------------|----------|-------------|--------------------|---------------|------------------|
|             |                                        |          | m           |                    | m             |                  |
| C O L U M N | 5 <sup>m</sup> . 8'                    | nice 21' | Astragal,   | — 2                | I             | 17 $\frac{1}{4}$ |
|             |                                        |          | Ove,        | — 5                | I <i>Sup.</i> | 21 $\frac{1}{4}$ |
|             |                                        |          | Goutiere,   | — 6 $\frac{1}{2}$  | I             | 25 $\frac{1}{4}$ |
|             |                                        |          | Reglet,     | — 2 $\frac{1}{2}$  | I             | 26 $\frac{1}{4}$ |
|             | Ionick Base 1 <sup>m</sup> .           |          | Plinthe,    | — 10               | I             | 11 $\frac{1}{4}$ |
|             |                                        |          | Orle,       | — 1 $\frac{1}{2}$  | I             | 10 $\frac{3}{4}$ |
|             |                                        |          | Scotie,     | — 3 $\frac{1}{2}$  | —             | —                |
|             |                                        |          | Orle 2.     | — 1 $\frac{1}{2}$  | I             | 6 $\frac{1}{2}$  |
|             |                                        |          | Astragal 1. | — 1 $\frac{1}{4}$  | I             | 7 $\frac{1}{4}$  |
|             |                                        |          | Astragal 2. | — 1 $\frac{1}{4}$  | —             | —                |
|             |                                        |          | Orle 3.     | — 1 $\frac{1}{2}$  | I             | 6 $\frac{1}{2}$  |
|             |                                        |          | Scotie,     | — 3 $\frac{1}{2}$  | —             | —                |
|             |                                        |          | Orle 4      | — 1 $\frac{1}{2}$  | I             | 3                |
|             |                                        |          | Tore,       | — 8 $\frac{1}{2}$  | I             | 7 $\frac{1}{4}$  |
|             | F&B 16 <sup>m</sup> . 11 $\frac{1}{3}$ |          | Reglet with | — 3                | I             | 3                |
|             |                                        |          | Chanfrain,  | — 3                | —             | —                |
|             |                                        |          | Trunk,      | 16 3 $\frac{2}{3}$ | above         | 26               |
|             |                                        |          | Orle above, | — 1 $\frac{1}{3}$  | I             | 2 $\frac{2}{3}$  |
|             |                                        |          | Astragal,   | — 3 $\frac{1}{3}$  | I             | 3 $\frac{1}{3}$  |

|                   |         |            |                     |   |                  |
|-------------------|---------|------------|---------------------|---|------------------|
| 16 <sup>m</sup> . | Capital | Ove,       | — † 6 $\frac{2}{3}$ | I | 5                |
|                   |         | Bande of   | — 5 $\frac{1}{3}$   | — | —                |
|                   |         | Volutes,   | — 1 $\frac{1}{3}$   | — | 28 $\frac{1}{3}$ |
|                   |         | Bordure of | — 3 $\frac{1}{3}$   | — | —                |

What follows must be referred to both Distributions.

| Parts.                            |                                  | Members.                              | Heights. |                                 | Ecphoræ. |                                |
|-----------------------------------|----------------------------------|---------------------------------------|----------|---------------------------------|----------|--------------------------------|
|                                   |                                  |                                       | m        |                                 | m        |                                |
| 11 <sup>2</sup> / <sub>3</sub> .  | 18 <sup>1</sup> / <sub>2</sub> . | Reglet,                               | —        | 1 <sup>2</sup> / <sub>3</sub>   | 1        | 1 <sup>2</sup> / <sub>3</sub>  |
|                                   |                                  | Perpendicu-<br>lar to Vo-<br>lutes,   | 1        | 1 <sup>2</sup> / <sub>3</sub>   | —        | 28 <sup>1</sup> / <sub>3</sub> |
|                                   |                                  | Perpendicu-<br>lar within<br>Volutes, | —        | 26 <sup>2</sup> / <sub>3</sub>  | —        | —                              |
| ENTABLEMENT 3 <sup>m</sup> . 18'. | Architrave 1 <sup>m</sup> . 16'. | Plat Bande 1.                         | —        | 6                               | —        | 27                             |
|                                   |                                  | Astragal,                             | —        | 1 <sup>1</sup> / <sub>2</sub>   | —        | 27 <sup>3</sup> / <sub>4</sub> |
|                                   |                                  | Plat Bande 2.                         | —        | 7 <sup>1</sup> / <sub>2</sub>   | —        | 27 <sup>3</sup> / <sub>4</sub> |
|                                   |                                  | Astragal,                             | —        | 2                               | —        | 28 <sup>3</sup> / <sub>4</sub> |
|                                   |                                  | Plat Bande<br>3.                      | 1        | 12                              | —        | 28 <sup>3</sup> / <sub>4</sub> |
|                                   |                                  | Talon,                                | —        | 5                               | —        | —                              |
|                                   | Cornice 1 <sup>m</sup> . 15'.    | Reglet,                               | —        | 2                               | 1        | 1 <sup>2</sup> / <sub>3</sub>  |
|                                   |                                  | Frize,                                | —        | 27                              | 1 *      | 1 <sup>2</sup> / <sub>3</sub>  |
|                                   |                                  | Cavett,                               | —        | 5                               | —        | —                              |
|                                   |                                  | Reglet,                               | —        | 1                               | 1        | 1 <sup>2</sup> / <sub>3</sub>  |
|                                   |                                  | Ove,                                  | —        | 6                               | 1 sup.   | 6                              |
|                                   |                                  | Bande,                                | —        | * 8 <sup>1</sup> / <sub>2</sub> | 1 *      | 21                             |
|                                   |                                  | Cymaife,                              | —        | † 3                             | 1 sup.   | 22 <sup>1</sup> / <sub>2</sub> |
|                                   |                                  | Goutiere,                             | —        | 8                               | 1        | 28 <sup>1</sup> / <sub>2</sub> |
|                                   |                                  | Talon,                                | —        | 3 <sup>1</sup> / <sub>2</sub>   | —        | —                              |
|                                   |                                  | Filet,                                | —        | 1                               | 2        | 3                              |
|                                   |                                  | Doucine,                              | —        | 7                               | —        | —                              |
|                                   |                                  | Reglet.                               | —        | 2                               | 2        | 11                             |

^ The



^ The Diameter of the Eye of the Volute is  $3' \frac{1}{3}$ . It has of Perpendicular above it  $13' \frac{1}{3}$ . and below it  $10'$ . The whole Breadth of the Volute is  $23'$ .

† On the Ove of the Capital are described Ovals, Leaves and Flowers on the Bande of Volutes, Olives on the Astragals, small Leaves on the Talon of the Abaque, and Roses on the Eyes of the Volutes.

\* The Frize terminates in the Arch of a Circle, having its Projection beyond the Ecphora of the superior Extremity of the Fust  $5' \frac{2}{3}$ . So the whole Ecphora of the Frize is  $1^m. 1' \frac{2}{3}$ .

\*\* On the Bande of the Cornice are the Modillions, high  $7' \frac{1}{2}$ . broad  $10'$ . and having a Metope  $21'$ . Below them is a Filet  $1'$  high, and having  $1^m. 21'$  of Ecphora.

‡ At about  $2'$  Distance from the Extremity of the Goutiere, there is a Scotie below joyning the Cymaife; and at  $4'$  Distance from the Extremity of the Bande of Modillions, on the lower Part of the Bande, there is a Cymaife joyning the upper Edge of the Filet, below the Modillions.

The

230 *A short Introduction*

*The IONICK ORDER according to SCAMOZZI.*

**T**His Architect gives  $17^m \frac{1}{2}$ . to the *Ionick* Column, with its Base and Capital; of that he allows  $\frac{2}{7}$ , or  $5^m$ . for the Pedestal, and  $\frac{1}{3}$ , or  $3^m. 15'$ . to the Entablement. So that, if the Frize has no Ornament, the whole Ordonnance is  $26^m$  high. But if the Nature of the Building requires Ornaments on the Frize, he then allows to the Entablement  $3^m. 26' \frac{2}{3}$ . and so the Altitude of the whole is  $26^m. 11' \frac{1}{3}$ .

| Parts.    |                            | Members.  | Heights.                      |                                | Ecphoræ.                       |                                |
|-----------|----------------------------|-----------|-------------------------------|--------------------------------|--------------------------------|--------------------------------|
|           |                            |           | m                             |                                | m                              |                                |
| PEDESTAL  | Base 1 <sup>m</sup> . 15'. | Socle,    | 1                             | —                              | 1                              | 24 <sup>3</sup> / <sub>4</sub> |
|           |                            | Tore,     | —                             | 3 <sup>1</sup> / <sub>4</sub>  | —                              | —                              |
|           |                            | Filet,    | —                             | <sup>3</sup> / <sub>4</sub>    | 1                              | 22 <sup>3</sup> / <sub>4</sub> |
|           |                            | Doucine,  | —                             | 5 <sup>1</sup> / <sub>4</sub>  | —                              | —                              |
|           |                            | Astragal, | —                             | 1 <sup>3</sup> / <sub>4</sub>  | 1 Cen.                         | 15 <sup>1</sup> / <sub>4</sub> |
|           |                            | Reglet,   | —                             | <sup>3</sup> / <sub>4</sub>    | 1                              | 15 <sup>1</sup> / <sub>4</sub> |
|           |                            | Cavett,   | —                             | 2 <sup>3</sup> / <sub>4</sub>  | —                              | —                              |
|           | Cor-                       | Dy,       | 2                             | 22 <sup>1</sup> / <sub>2</sub> | 1                              | 11 <sup>3</sup> / <sub>4</sub> |
|           |                            | Cavett,   | —                             | 4 <sup>1</sup> / <sub>4</sub>  | —                              | —                              |
|           |                            | Filet,    | —                             | 1 <sup>1</sup> / <sub>4</sub>  | 1                              | 15 <sup>1</sup> / <sub>4</sub> |
| Astragal, |                            | —         | 1 <sup>3</sup> / <sub>4</sub> | 1 Cen.                         | 15 <sup>1</sup> / <sub>4</sub> |                                |

Parts.

| Parts.                                     |                                              | Members.      | Heighrs.             | Ecphoræ.               |
|--------------------------------------------|----------------------------------------------|---------------|----------------------|------------------------|
|                                            |                                              |               | m                    | m                      |
| COLUMN 17 <sup>m</sup> . 15 <sup>f</sup> . | 5 <sup>m</sup> .<br>nice 22 <sup>f</sup> .   | Ove,          | — 5                  | I <i>sup.</i> 19       |
|                                            |                                              | Goutiere,     | — * 4 $\frac{1}{2}$  | I * 23 $\frac{3}{4}$   |
|                                            |                                              | Talon,        | — 3 $\frac{1}{2}$    | —                      |
|                                            |                                              | Reglet,       | — 2 $\frac{1}{4}$    | — 27 $\frac{1}{4}$     |
|                                            | Base 1 <sup>m</sup> .                        | Plinthe,      | — 10 $\frac{2}{3}$   | I 11 $\frac{3}{4}$     |
|                                            |                                              | Tore below,   | — 8                  | I Cen. 7 $\frac{1}{4}$ |
|                                            |                                              | Orle 1.       | — 3 $\frac{3}{4}$    | I 7 $\frac{1}{4}$      |
|                                            |                                              | Scotie,       | — 4                  | —                      |
|                                            |                                              | Orle 2.       | — 1 $\frac{1}{4}$    | I 5 $\frac{1}{4}$      |
|                                            |                                              | Tore above,   | — 5 $\frac{1}{3}$    | I 7 $\frac{3}{4}$      |
|                                            | Fust 15 <sup>m</sup> . 26 <sup>f</sup> .     | Astragal,     | — 2 $\frac{1}{3}$    | I 5 $\frac{3}{4}$      |
|                                            |                                              | Orle w Cav.   | — 1 $\frac{1}{3}$    | I 4 $\frac{5}{12}$     |
|                                            |                                              | Trunk,        | 15 17 $\frac{7}{12}$ | above 25               |
|                                            |                                              | Orle with     | — 1 $\frac{1}{3}$    | — 27 $\frac{1}{3}$     |
|                                            | Capital 18 <sup>f</sup> .                    | Conge,        | — 3 $\frac{1}{3}$    | — 29                   |
|                                            |                                              | Astragal,     | — 6 $\frac{2}{3}$    | I 3 $\frac{1}{3}$      |
|                                            |                                              | Ove,          | — 5                  | — 28 $\frac{1}{3}$     |
|                                            |                                              | Bande of      | — 1 $\frac{1}{3}$    | I —                    |
|                                            |                                              | Volutes,      | — 3 $\frac{1}{2}$    | —                      |
|                                            |                                              | Bordure,      | — 2 $\frac{1}{4}$    | I 1 $\frac{2}{3}$      |
|                                            |                                              | Talon,        | —                    | —                      |
|                                            |                                              | Reglet,       | —                    | —                      |
|                                            | Architrave 1 <sup>m</sup> . 5 <sup>f</sup> . | Plat Bande,   | — 7                  | — 25                   |
|                                            |                                              | Plat Bande 2. | — 8 $\frac{3}{4}$    | — 26                   |
|                                            |                                              | Plat Bande 3. | — 10                 | — 27                   |
|                                            |                                              | Astragal,     | — 1 $\frac{3}{4}$    | — 29                   |
|                                            |                                              | Talon,        | — 4 $\frac{1}{2}$    | —                      |
|                                            |                                              | Reglet,       | — 3                  | I 1 $\frac{2}{3}$      |

\* At the Ex-  
tremity of  
the Gou-  
tiere there  
is a Scotie  
that joyns  
the Ove  
below.



| Parts.                          |                               | Members.   | Heights. |                    | Ecphora. |                  |
|---------------------------------|-------------------------------|------------|----------|--------------------|----------|------------------|
|                                 |                               |            | m        | '                  | m        | '                |
| TABLEMENT 3 <sup>m</sup> . 15'. | L'vize 28'.                   | Plat Bande | —        | † 26 $\frac{1}{2}$ | —        | 25               |
|                                 |                               | Reglet,    | —        | 1 $\frac{1}{2}$    | —        | 27               |
|                                 | Cornice 1 <sup>m</sup> . 12'. | Talon,     | —        | 4                  | —        | —                |
|                                 |                               | Filet,     | —        | 1                  | 1        | 1                |
|                                 |                               | Goutiere,  | —        | 4 $\frac{2}{3}$    | 1        | 5 $\frac{2}{3}$  |
|                                 |                               | Filet,     | —        | 1                  | 1        | 6                |
|                                 |                               | Ove,       | —        | 4                  | —        | —                |
|                                 |                               | Bande of   | —        | *                  | 1        | *                |
|                                 |                               | Modillons, | —        | 7                  | 1        | 11 $\frac{1}{8}$ |
|                                 |                               | Cymaife of | —        | 2 $\frac{1}{3}$    | —        | —                |
|                                 |                               | Modillons, | —        | 2 $\frac{1}{3}$    | —        | —                |
|                                 |                               | Goutiere,  | —        | 6                  | 2        | 2 $\frac{1}{2}$  |
|                                 |                               | Cymaife,   | —        | 3                  | —        | —                |
|                                 |                               | Filet,     | —        | 1                  | 2        | 5 $\frac{5}{8}$  |
|                                 |                               | Gueule,    | —        | 6                  | —        | —                |
|                                 |                               | Reglet.    | —        | 2                  | 2        | 11 $\frac{1}{2}$ |

\* \* To the Bande of Modillons is  
 \* joined a Modillon, in Breadth 19'.  
 and 4' of these on the outward Part  
 are plain, the other 15'. are turned  
 spirally. So the whole Bande, inclu-  
 ding that Modillon, has 2<sup>m</sup>  $\frac{1}{8}$ ' of Ec-  
 phora.

The Breadth of the other Modil-  
 lons is 11  $\frac{2}{3}$ '. their Height 7'. and their  
 Metope 23'  $\frac{1}{3}$ .

In

In our Definitions of the Five Orders of Architecture, we put the *Corinthian* Order in the last Place, as being the most delicate and beautiful of all the Orders, yet, because it is before the *Composite* in the Order of Time, we shall here lay down the Proportions of its Parts, and their Members, in the Fourth Place, leaving the *Composite* to the last.

*The CORINTHIAN ORDER according to PALLADIO.*

**T**His Author gives to the *Corinthian* Column, including the Base and Capital,  $19^m$ . viz. to the Base  $1^m$ . to the Fust  $15^m. 20'$ . and to the Capital  $2^m. 10'$ . To the Pedestal he allows  $\frac{1}{4}$  of that, viz.  $4^m. 22'\frac{1}{2}$ . of which he gives  $1^m. 6'$ . to the Base,  $2^m. 28'\frac{1}{2}$ . to the Dy, and  $18'$ . to the Cornice.

He gives  $\frac{1}{3}$  the Height of the Column, i. e.  $3^m. 24'$ . to the Entablement, of which he allows  $1^m. 8'$ . to the Architrave,  $28'\frac{1}{2}$ . to the Frize, and  $1^m. 17'\frac{1}{2}$ . to the Cornice. So that, according

G g

# 234 *A short Introduction*

ding to this Distribution of the Parts, the whole Order is in Height 27<sup>m</sup>. 16'<sup>1</sup>/<sub>2</sub>. tho', in the Detail of the Members of the several Parts, he differs a little from this Disposition, allowing, for Instance, 5<sup>m</sup>. to the Pedestal, 3<sup>m</sup>. 25'. to the Entablement, &c.

| Parts.   |                                                        | Members.    | Heights. |                                | Ecphoræ.      |                                |
|----------|--------------------------------------------------------|-------------|----------|--------------------------------|---------------|--------------------------------|
|          |                                                        |             | m        | '                              | m             | '                              |
| PEDESTAL | Base 1 <sup>m</sup> . 7' <sup>1</sup> / <sub>2</sub> . | Sccle,      | —        | 23 <sup>1</sup> / <sub>2</sub> | I             | 27                             |
|          |                                                        | Tore,       | —        | 4                              | —             | —                              |
|          |                                                        | Filet,      | —        | <sup>1</sup> / <sub>4</sub>    | I             | 25                             |
|          |                                                        | Gueule inv. | —        | 5                              | —             | —                              |
|          |                                                        | Reglet,     | —        | <sup>3</sup> / <sub>4</sub>    | I             | 17                             |
|          |                                                        | Talon,      | —        | 4                              | I <i>sup.</i> | 12 <sup>1</sup> / <sub>2</sub> |
|          | Cornice 19'.                                           | Dy,         | 3        | 3 <sup>1</sup> / <sub>2</sub>  | I             | 12                             |
|          |                                                        | Talon,      | —        | 3 <sup>3</sup> / <sub>4</sub>  | —             | —                              |
|          |                                                        | Reglet,     | —        | <sup>5</sup> / <sub>4</sub>    | I             | 16                             |
|          |                                                        | Ove,        | —        | 4 <sup>1</sup> / <sub>4</sub>  | — *           | —                              |
|          |                                                        | Goutiere,   | —        | 4 <sup>1</sup> / <sub>4</sub>  | I *           | 23 <sup>1</sup> / <sub>4</sub> |
|          |                                                        | Talon,      | —        | 3 <sup>1</sup> / <sub>2</sub>  | —             | —                              |
|          |                                                        | Reglet,     | —        | 2 <sup>1</sup> / <sub>2</sub>  | I             | 27                             |
|          |                                                        | Plinthe,    | —        | 9 <sup>2</sup> / <sub>3</sub>  | I             | 12                             |
|          |                                                        | Tore below, | —        | 7                              | —             | —                              |

\*\* In the Goutiere at 3'<sup>3</sup>/<sub>4</sub>. from its Extremity is a Scotie joyning the Ove.

Parts.



| Parts.                   | Members.                                   | Heights.     |    | Ecphoræ.        |                 |
|--------------------------|--------------------------------------------|--------------|----|-----------------|-----------------|
|                          |                                            | m            | '  | m               | '               |
| COLUMN 19 <sup>m</sup> . | Base 1 <sup>m</sup> .                      | Astragal,    | —  | 1 $\frac{3}{4}$ | 8 $\frac{1}{2}$ |
|                          |                                            | Filet,       | —  | 1 $\frac{3}{4}$ | 7 $\frac{7}{8}$ |
|                          |                                            | Scotie,      | —  | 3 $\frac{1}{2}$ | —               |
|                          |                                            | Orle,        | —  | 1 $\frac{2}{3}$ | 4 $\frac{2}{3}$ |
|                          |                                            | Astragal,    | —  | 1 $\frac{2}{3}$ | 5 $\frac{1}{2}$ |
|                          |                                            | Tore,        | —  | 5               | 7 $\frac{1}{2}$ |
|                          | Fust 15 <sup>m</sup> . 20 <sup>o</sup> .   | Astragal,    | —  | 2 $\frac{1}{2}$ | 5 $\frac{1}{2}$ |
|                          |                                            | Orle with    | —  | 1 $\frac{1}{4}$ | 4               |
|                          |                                            | Conge,       | 15 | 11              | above 26        |
|                          |                                            | Trunk,       | —  | 1 $\frac{1}{2}$ | 28              |
|                          |                                            | Orle with    | —  | 1 $\frac{1}{2}$ | —               |
|                          |                                            | Conge,       | —  | *               | 3 $\frac{1}{4}$ |
|                          |                                            | Astragal,    | —  | 3 $\frac{1}{4}$ | 1               |
|                          | Capital 2 <sup>m</sup> . 10 <sup>o</sup> . | Lower        | —  | 15              | —               |
|                          |                                            | Leaves,      | —  | 5               | —               |
|                          |                                            | Their Fol-   | —  | —               | —               |
|                          |                                            | ding,        | —  | —               | —               |
|                          |                                            | Middle       | —  | 15              | —               |
|                          |                                            | Leaves,      | —  | 5               | —               |
|                          |                                            | Their Fol-   | —  | —               | —               |
|                          |                                            | ding,        | —  | —               | —               |
|                          |                                            | Leaves a-    | —  | 10              | —               |
|                          |                                            | bove,        | —  | 7 $\frac{1}{2}$ | —               |
|                          |                                            | Helices,     | —  | 2 $\frac{1}{2}$ | 1               |
|                          |                                            | Orle of Vef- | —  | 2 $\frac{1}{2}$ | 3               |
|                          |                                            | fel,         | —  | 5 $\frac{2}{3}$ | x               |
|                          |                                            | Plinthe,     | —  | 1 $\frac{1}{3}$ | *               |
|                          |                                            | Reglet,      | —  | *               | x               |
|                          |                                            | Ove,         | —  | 3               | —               |

| Parts.                                                    | Members.      | Heights. |                                | Ecphoræ. |                                |
|-----------------------------------------------------------|---------------|----------|--------------------------------|----------|--------------------------------|
|                                                           |               | m        | '                              | m        | '                              |
| Architrave 1 <sup>m</sup> . 8 <sup>1</sup> / <sub>4</sub> | Plat Bande 1. | —        | 6 <sup>1</sup> / <sub>4</sub>  | —        | 26                             |
|                                                           | Filet, Cord,  | —        | 1 <sup>3</sup> / <sub>4</sub>  | —        | 27                             |
|                                                           | Plat Bande 2. | —        | 8 <sup>1</sup> / <sub>4</sub>  | —        | 27                             |
|                                                           | Astragal,     | —        | 2 <sup>1</sup> / <sub>4</sub>  | —        | 28                             |
|                                                           | Plat Bande    | —        | 10 <sup>1</sup> / <sub>2</sub> | —        | 28                             |
|                                                           | 3.            | —        | —                              | —        | —                              |
|                                                           | Astragal,     | —        | 2                              | —        | 29 <sup>1</sup> / <sub>4</sub> |
|                                                           | Talon,        | —        | 5                              | below    | 28 <sup>1</sup> / <sub>4</sub> |
|                                                           | Reglet,       | —        | 2 <sup>3</sup> / <sub>4</sub>  | 1        | 4 <sup>1</sup> / <sub>2</sub>  |
|                                                           | Frize, ‡      | —        | 28 <sup>1</sup> / <sub>2</sub> | —        | 26                             |
| Cornice 1 <sup>m</sup> . 17 <sup>1</sup> / <sub>2</sub>   | Talon,        | —        | 4 <sup>1</sup> / <sub>2</sub>  | —        | —                              |
|                                                           | Reglet,       | —        | 1                              | 1        | 1 <sup>1</sup> / <sub>4</sub>  |
|                                                           | Bande of      | —        | —                              | —        | —                              |
|                                                           | Denticules,   | —        | 5 <sup>1</sup> / <sub>2</sub>  | —        | —                              |
|                                                           | Reglet,       | —        | 1                              | 1        | 5                              |
|                                                           | Ove,          | —        | 4 <sup>1</sup> / <sub>2</sub>  | —        | —                              |
|                                                           | Filet under   | —        | 1                              | 1        | 11                             |
|                                                           | Modillons,    | —        | —                              | —        | —                              |
|                                                           | Bande of      | —        | 7 <sup>1</sup> / <sub>2</sub>  | *        | *                              |
|                                                           | Modillons,    | —        | —                              | —        | —                              |
|                                                           | Talon,        | —        | 2 <sup>1</sup> / <sub>3</sub>  | —        | —                              |
|                                                           | Filet,        | —        | —                              | 2        | 2                              |
|                                                           | Goutiere,     | —        | 7 <sup>1</sup> / <sub>3</sub>  | 2        | 2 <sup>1</sup> / <sub>4</sub>  |
|                                                           | Talon,        | —        | 3                              | —        | —                              |
|                                                           | Filet,        | —        | —                              | 2        | 6                              |
|                                                           | Doucine,      | —        | 6 <sup>1</sup> / <sub>2</sub>  | x        | x                              |
|                                                           | Reglet.       | —        | 2 <sup>1</sup> / <sub>4</sub>  | 2        | 13 <sup>1</sup> / <sub>2</sub> |

‡ Joyned  
with an  
Arch to  
the Reglet  
of the Ar-  
chitrave.

x The

x The *Ecphoræ* of the Plinthe, Reglet and Ove, together with the Projection of the Leaves, are determined thus.

Describe a Square  $ABCD$ , whose Plate XIX.  
Side  $CD = 3^m$ . draw the Diameters  $AC$ ,  $BD$ , on  $BD$ , set off  $Et = 2^m = ES$ , through  $t$  and  $s$ , draw  $KL$  and  $FG \parallel AC$ , and continue them; take  $tm = 4\frac{1}{2}$ ,  $tn = 6\frac{1}{2}$ , and draw  $mx$ ,  $nz \parallel KL$ , so  $nz$  limits the Plinthe,  $mx$  the Reglet, and  $to$  the Ove, draw  $OR$  joyning the Extremities of the Ove and Astragal, that  $RO$  limits the Projection of the Leaves.

On  $VL$  raise an equilateral Triangle, on its Vertex as a Center, with one of its Sides as a Radius, describe the Arch  $Lxv$  for the Limit of the Front of the Capital.

The like Construction may be made on  $pQ$ , parallel to one of the Sides  $CD$ , which restricts the Capital and Leaves to narrower Bounds than the other; for, in that the *Ecphora* of the Ove is  $2^m$ . and in this only  $1^m$ .  $15'$ . This last Construction Mons<sup>r</sup>.  
*Blon-*



*Blondel* suspects to be meant by *Paladio*, that Architect being a little obscure in this Matter.

The Spirals or Volutes in the Middle just touch the Orle of the Vessel, as is plain from the Table of Heights and *Ecphoræ*; but the Volutes on the Corners not only possess all the Space that the middle Volutes have, and the Orle of the Vessel which is 10'. but likewise dip a little in the Plinthe or Goutiere of the Abaque.

The Rose in the Middle possesses the whole Altitude of the Abaque, and that of the Orle of the Vessel.

If the Column is to be canalled, it must have Twenty four Canals, hollowed each to a Semicircle, and the Breadth of each Listel or Interstice must be  $\frac{1}{3}$  the Diameter of the Canal.

The Bottom of the Vessel must answer directly to the Hollow of the Canals.

The Breadth of each Modillon is 12'. and their Metope  $23\frac{1}{4}$ .

\* In the Bande of Modillons, betwixt the Extremity of the Reglet  
above

above the Modillions, and that of the Filet under the Modillions, is a spiral Modillon of  $21\frac{1}{4}$  Breadth.

*The CORINTHIAN ORDER according to SCAMOZZI.*

**T**HIS Architect gives to the *Corinthian* Column, with its Base and Capital,  $20^m$ . which he says is the greatest Height a Pillar can have; the Base is  $1^m$ . the Fust  $16^m$ .  $20'$ . and the Capital  $2^m$ .  $10'$ .

Of this Height of the Column, *viz.*  $20^m$ . he allows  $\frac{1}{3}$ , or  $6^m$ .  $20'$ . to the Pedestal, giving  $1^m$ .  $15'$ . to the Base,  $4^m$ .  $12\frac{1}{2}'$ . to the Trunk or Dy, and  $22\frac{1}{2}'$ . to the Cornice.

He gives  $\frac{1}{3}$  the Altitude of the Column, *i. e.*  $4^m$ . to the Entablement, allowing  $1^m$ .  $10'$ . to the Architrave,  $1^m$ .  $2'$ . to the Frize, and  $1^m$ .  $18'$ . to the Architrave, in case there are no Ornaments on the Frize; but if the Frize has Ornaments, its Altitude must then be  $1^m$ .  $15\frac{1}{3}'$ . and then the whole Entablement has  $4^m$ .  $13\frac{1}{3}'$ . of Altitude.

Thus

# 240 *A Short Introduction*

Thus when the Architrave has no Ornament, the Altitude of the whole Order is 30<sup>m</sup>. 20'. but if the Frize has Ornaments, the Altitude of the whole is 31<sup>m</sup>. 3'  $\frac{1}{3}$ .

| Parts.                         | Members.                                                                    | Heights.              |   | Ecphora.        |                                                                                                                      |
|--------------------------------|-----------------------------------------------------------------------------|-----------------------|---|-----------------|----------------------------------------------------------------------------------------------------------------------|
|                                |                                                                             | m                     | ' | m               | '                                                                                                                    |
| PEDESTAL 6 <sup>m</sup> . 20'. | Base 1 <sup>m</sup> . 15'.                                                  | Socle,                | 1 | —               | 1 21 $\frac{1}{2}$                                                                                                   |
|                                |                                                                             | Tore,                 | — | 2 $\frac{2}{3}$ | —                                                                                                                    |
|                                |                                                                             | Reglet,               | — | 1 $\frac{1}{3}$ | 20                                                                                                                   |
|                                |                                                                             | Gueule inv.           | — | 4               | —                                                                                                                    |
|                                |                                                                             | Reglet,               | — | 1 $\frac{1}{3}$ | 14                                                                                                                   |
|                                |                                                                             | Scotie,               | — | 2 $\frac{1}{3}$ | —                                                                                                                    |
|                                |                                                                             | Reglet,               | — | 1 $\frac{1}{3}$ | 13 $\frac{1}{2}$                                                                                                     |
|                                |                                                                             | Tore,                 | — | 2               | 14                                                                                                                   |
|                                | Trunk H. 4 <sup>m</sup> . 12' $\frac{1}{2}$ .<br>Ec. 1. 11' $\frac{1}{4}$ . | Reglet with<br>Conge, | — | 1               | 12 $\frac{5}{8}$                                                                                                     |
|                                |                                                                             | Bande,                | — | 7 $\frac{3}{4}$ | The Talon with<br>its Filets sur-<br>round the whole<br>Dy, leaving on<br>all Sides a Bande<br>of 7' $\frac{3}{4}$ . |
|                                |                                                                             | Filet,                | — | 3 $\frac{3}{4}$ |                                                                                                                      |
|                                |                                                                             | Talon,                | — | 2               |                                                                                                                      |
|                                |                                                                             | Filet,                | — | 3 $\frac{3}{4}$ |                                                                                                                      |
|                                | Cornice                                                                     | Talon,                | — | 3 $\frac{1}{4}$ | —                                                                                                                    |
|                                |                                                                             | Filet,                | — | 1               | 15 $\frac{11}{12}$                                                                                                   |
|                                |                                                                             | Astragal,             | — | 1 $\frac{1}{4}$ | 1 Cen. 15 $\frac{11}{12}$                                                                                            |
|                                |                                                                             | Ove,                  | — | 4 $\frac{2}{3}$ | 1 inf. 15 $\frac{11}{12}$                                                                                            |
|                                |                                                                             | Reglet,               | — | 1               | 18                                                                                                                   |
|                                |                                                                             | Goutiere,             | — | 4 $\frac{1}{3}$ | 1 * 24 $\frac{1}{4}$                                                                                                 |

Parts.



| Parts.                   |                                          | Members.                            | Heights. |                                | Ecphoræ.                             |                                |
|--------------------------|------------------------------------------|-------------------------------------|----------|--------------------------------|--------------------------------------|--------------------------------|
|                          |                                          |                                     | m        | '                              | m                                    | '                              |
| COLUMN 20 <sup>m</sup> . | 22 <sup>1</sup> / <sub>2</sub> .         | Filet,                              | —        | 1 <sup>1</sup> / <sub>4</sub>  | —                                    | —                              |
|                          |                                          | Talon,                              | —        | 3                              | —                                    | —                              |
|                          |                                          | Reglet,                             | —        | 2                              | 1                                    | 27 <sup>1</sup> / <sub>4</sub> |
|                          | Base 1 <sup>m</sup> .                    | Plinthe,                            | —        | 9 <sup>1</sup> / <sub>3</sub>  | 1                                    | 11 <sup>1</sup> / <sub>4</sub> |
|                          |                                          | Tore,                               | —        | 7                              | —                                    | —                              |
|                          |                                          | Astragal,                           | —        | 2                              | 1                                    | 8                              |
|                          |                                          | Orle,                               | —        | 1                              | 1                                    | 6 <sup>7</sup> / <sub>12</sub> |
|                          |                                          | Scotie,                             | —        | 3 <sup>1</sup> / <sub>2</sub>  | —                                    | —                              |
|                          |                                          | Orle,                               | —        | 1                              | 1                                    | 5 <sup>1</sup> / <sub>4</sub>  |
|                          |                                          | Astragal,                           | —        | 1 <sup>1</sup> / <sub>2</sub>  | 1                                    | 5 <sup>7</sup> / <sub>12</sub> |
|                          |                                          | Tore,                               | —        | 4 <sup>1</sup> / <sub>3</sub>  | 1                                    | 8                              |
|                          | Fust 16 <sup>m</sup> . 20 <sup>o</sup> . | Astragal be-<br>low,                | —        | 2 <sup>1</sup> / <sub>2</sub>  | 1                                    | 5 <sup>7</sup> / <sub>12</sub> |
|                          |                                          | Orle with<br>Chanfrain,             | —        | 1 <sup>1</sup> / <sub>3</sub>  | 1                                    | 4 <sup>1</sup> / <sub>4</sub>  |
|                          |                                          | Trunk,                              | 16       | 11 <sup>2</sup> / <sub>3</sub> | above 26 <sup>1</sup> / <sub>2</sub> | —                              |
|                          |                                          | Orle above,<br>with Chan-<br>frain, | —        | 1 <sup>2</sup> / <sub>3</sub>  | —                                    | 27                             |
|                          |                                          | Astragal,                           | —        | 3 <sup>1</sup> / <sub>2</sub>  | —                                    | 29 <sup>1</sup> / <sub>2</sub> |
|                          | Capital                                  | Leaves 1.                           | —        | 15                             | —                                    | —                              |
|                          |                                          | Their Folds,                        | —        | 5                              | —                                    | —                              |
|                          |                                          | Leaves 2.                           | —        | 15                             | —                                    | —                              |
|                          |                                          | Their Folds,                        | —        | 5                              | —                                    | —                              |
|                          |                                          | Leaves 3.                           | —        | 10                             | —                                    | —                              |
|                          |                                          | Spirals,                            | —        | 7 <sup>1</sup> / <sub>2</sub>  | —                                    | —                              |

Joyned  
with an  
Arch to  
the Reglet  
below.

| Parts.                       |                                               | Members.                     | Heights.        |                   | Ecphoræ. |                   |
|------------------------------|-----------------------------------------------|------------------------------|-----------------|-------------------|----------|-------------------|
|                              |                                               |                              | m               |                   | m        |                   |
| ENTABLEMENT 4 <sup>m</sup> . | 2 <sup>m</sup> . 10 <sup>o</sup> .            | Orle of Vessel,              | —               | 2 $\frac{1}{2}$   | —        | —                 |
|                              |                                               | Goutiere,                    | —               | 6                 | —        | —                 |
|                              |                                               | Reglet,                      | —               | 1                 | —        | —                 |
|                              |                                               | Ove,                         | —               | 3                 | —        | —                 |
|                              | Architrave 1 <sup>m</sup> . 10 <sup>o</sup> . | Plat Bande 1.                | —               | 6 $\frac{1}{3}$   | —        | 26 $\frac{1}{2}$  |
|                              |                                               | Astragal,                    | —               | 1 $\frac{1}{3}$   | —        | 27 $\frac{1}{3}$  |
|                              |                                               | Bande 2.                     | —               | 8 $\frac{1}{3}$   | —        | 27 $\frac{1}{3}$  |
|                              |                                               | Talon,                       | —               | 2 $\frac{1}{4}$   | —        | —                 |
|                              |                                               | Bande 3.                     | —               | 11 $\frac{1}{4}$  | —        | 29 $\frac{5}{8}$  |
|                              |                                               | Cord, or Astragal,           | —               | 1 $\frac{1}{3}$   | 1        | 2 $\frac{1}{4}$   |
|                              |                                               | Talon,                       | —               | 2 $\frac{3}{4}$   | 1 inf.   | 2 $\frac{1}{4}$   |
|                              |                                               | Chanfrain,                   | —               | 3 $\frac{1}{3}$   | 1 inf.   | 3 $\frac{2}{3}$   |
|                              |                                               | Reglet,                      | —               | 2                 | 1        | 5 $\frac{1}{3}$   |
|                              |                                               | Frize,                       | 1 $\frac{1}{2}$ | 2 $\frac{1}{3}$   | —        | † 26              |
| Cornice                      |                                               | Talon,                       | —               | 4 $\frac{1}{2}$   | —        | —                 |
|                              |                                               | Filet,                       | —               | 1                 | 1        | 3 $\frac{1}{3}$   |
|                              |                                               | Astragal,                    | —               | 1 $\frac{2}{3}$   | 1        | 3 $\frac{1}{3}$   |
|                              |                                               | Ove,                         | —               | 5                 | —        | —                 |
|                              |                                               | Reglet under the Modillions, | —               | 1 $\frac{1}{3}$   | 1        | 5 $\frac{1}{4}$   |
|                              |                                               | Bande of Modillions,         | —               | 8 $\frac{1}{3}$   | 1 +      | 22 $\frac{7}{12}$ |
|                              |                                               | Talon,                       | —               | 2 $\frac{3}{4}$   | —        | —                 |
|                              |                                               | Reglet,                      | —               | † 1 $\frac{1}{3}$ | 2        | 1 $\frac{1}{3}$   |

† If not adorned.  
 \* With Ornaments.  
 ‡ Joyned with an Arch to the Reglet of the Architrave.

Parts.

| Parts.               | Members   | Heights.          | Ecphoræ.           |
|----------------------|-----------|-------------------|--------------------|
|                      |           | m                 | m                  |
| 1 <sup>m</sup> . 18. | Goutiere, | — 7 $\frac{1}{3}$ | 2 2 $\frac{1}{2}$  |
|                      | Cord,     | — 1 $\frac{1}{3}$ | — —                |
|                      | Talon,    | — 3 $\frac{1}{3}$ | — —                |
|                      | Filet,    | — 1               | 2 6 $\frac{1}{2}$  |
|                      | Doucine,  | — 6 $\frac{2}{3}$ | — —                |
|                      | Reglet.   | — 2 $\frac{1}{4}$ | 2 14 $\frac{1}{2}$ |

† Allow 4'. on the End of the Reglet for a Modillon, and a blank of 6'  $\frac{3}{4}$ . So you have the Extremity of the higher Side of the Talon below the Reglet.

† On the exterior Part of the Bande of Modillons is a Modillon cut spiral at both Extremities, in Breadth 17'  $\frac{1}{3}$ .

The Volutes and Spirals are situated as in *Palladio's Corinthian Order*, and the Projections of the Leaves and Abaque are found after the same Manner as there.



*The ROMAN or COMPOSITE ORDER  
according to PALLADIO.*

**T**He Column, with its Base and Capital, according to this Architect, has 20<sup>m</sup>. of Height; of that he gives 1<sup>m</sup>. to the Base, 16<sup>m</sup>. 20'. to the Fust, and 2<sup>m</sup>. 10'. to the Capital.

The Height of the Pedestal is  $\frac{1}{3}$ , that of the Column, *i. e.* 6<sup>m</sup>. 20'. of which the Base has 1<sup>m</sup>. 18'. the Trunk or Dy 4<sup>m</sup>. 8'. and the Cornice 24'.

The Entablement has  $\frac{1}{5}$ , the Height of the Column, *i. e.* 4<sup>m</sup>. of which the Architrave has 1<sup>m</sup>. 10'. the Frize 1<sup>m</sup>. and the Cornice 1<sup>m</sup>. 20'. So the Height of the whole Order is 30<sup>m</sup>. 20'.

| Parts. |                            | Members.    | Heights. |                 | Ecphoræ. |                  |
|--------|----------------------------|-------------|----------|-----------------|----------|------------------|
|        |                            |             | m        | '               | m        | '                |
| PE-    | Base 1 <sup>m</sup> . 18'. | Socle,      | 1        | 2               | 1        | 27               |
|        |                            | Tore,       | —        | 4 $\frac{1}{2}$ | —        | —                |
|        |                            | Filet,      | —        | 1               | 1        | 24 $\frac{3}{4}$ |
|        |                            | Gueule inv. | —        | 7 $\frac{1}{2}$ | —        | —                |
|        |                            | Tore,       | —        | 3               | 1        | 18 $\frac{3}{4}$ |

Parts.

| Parts.      |                            | Members.                          | Heights. |                                | Ecphoræ.                      |                                |    |
|-------------|----------------------------|-----------------------------------|----------|--------------------------------|-------------------------------|--------------------------------|----|
|             |                            |                                   | m        | '                              | m                             | '                              |    |
| DESTAL      | Trunk 4 <sup>m</sup> . 8'. | Reglet with                       | —        | 1                              | 1                             | 15 <sup>1</sup> / <sub>2</sub> |    |
|             |                            | Conge,                            | 4        | 6                              | 1                             | 12                             |    |
|             |                            | Dy,                               | —        | 1                              | 1                             | 15 <sup>1</sup> / <sub>2</sub> |    |
|             | Cornice 24'.               | Reglet with                       | —        | 1                              | 1                             | 15 <sup>1</sup> / <sub>2</sub> |    |
|             |                            | Conge,                            | —        | 3                              | 1 Cen.                        | 14 <sup>3</sup> / <sub>4</sub> |    |
|             |                            | Tore,                             | —        | 8 <sup>1</sup> / <sub>2</sub>  | —                             | —                              |    |
|             |                            | Doucine,                          | —        | 1                              | 1                             | 22 <sup>1</sup> / <sub>2</sub> |    |
|             |                            | Reglet,                           | —        | 5 <sup>1</sup> / <sub>2</sub>  | 1                             | 23 <sup>1</sup> / <sub>2</sub> |    |
|             |                            | Goutiere,                         | —        | 3 <sup>1</sup> / <sub>2</sub>  | —                             | —                              |    |
|             | Co-                        | * Composite Base 1 <sup>m</sup> . | Talon,   | —                              | 2 <sup>1</sup> / <sub>2</sub> | 1                              | 27 |
|             |                            |                                   | Reglet,  | —                              | 9                             | 1                              | 12 |
|             |                            |                                   | Plinthe, | —                              | 7                             | —                              | —  |
| Tore,       |                            |                                   | —        | 1 <sup>1</sup> / <sub>4</sub>  | 1                             | 8 <sup>1</sup> / <sub>2</sub>  |    |
| Filet,      |                            |                                   | —        | 3                              | —                             | —                              |    |
| Scotie,     |                            |                                   | —        | 1 <sup>1</sup> / <sub>4</sub>  | 1                             | 6 <sup>1</sup> / <sub>2</sub>  |    |
| Filet,      |                            |                                   | —        | 1                              | 1                             | 7                              |    |
| Astragal,   |                            |                                   | —        | 1                              | 1                             | 7                              |    |
| Astragal 2. |                            |                                   | —        | 1 <sup>1</sup> / <sub>4</sub>  | 1                             | 6 <sup>1</sup> / <sub>2</sub>  |    |
| Filet,      |                            |                                   | —        | 3                              | —                             | —                              |    |
| Scotie,     |                            |                                   | —        | 1 <sup>1</sup> / <sub>4</sub>  | 1                             | 4 <sup>3</sup> / <sub>4</sub>  |    |
| Filet,      |                            |                                   | —        | 4 <sup>1</sup> / <sub>2</sub>  | 1                             | 7                              |    |
| Fuß         |                            | Tore,                             | —        | 3                              | 1                             | 4                              |    |
|             |                            | Astragal, †                       | —        | 1                              | 1                             | 2 <sup>1</sup> / <sub>2</sub>  |    |
|             |                            | Orle with                         | 16       | 10 <sup>1</sup> / <sub>2</sub> | above 26                      |                                |    |
|             |                            | Chanfrain,                        |          |                                |                               |                                |    |
|             |                            | Trunk,                            |          |                                |                               |                                |    |

\* The Base of the Column may be either *Attick*, as in the *Corinthian*, or composed of the *Attick* and *Ionick*, as here.

† The Capitals are Twenty four, as in the *Corinthian Order*.

Parts.

‡ The Capital (as in the *Corinthian*) consists of the Vessel 2<sup>m</sup>. and Abaque 10'. The Profil is the same as in the *Corinthian* Order.

| Parts.                 | Members.                         | Heights.                       |                   | Ecphora.           |                   |
|------------------------|----------------------------------|--------------------------------|-------------------|--------------------|-------------------|
|                        |                                  | m                              | '                 | m                  | '                 |
| LUMN 20 <sup>m</sup> . | 16 <sup>m</sup> . 20'.           | Orle with Chanfrain, Astragal, | — 1 $\frac{1}{2}$ | — 29 $\frac{1}{4}$ |                   |
|                        |                                  |                                | — 4               | 1                  | —                 |
|                        | ‡ Capital 2 <sup>m</sup> . 10'.  | Leaves 1.                      | — 15              | —                  | —                 |
|                        |                                  | Their Folds,                   | — 5               | —                  | —                 |
|                        |                                  | Leaves 2.                      | — 15              | —                  | —                 |
|                        |                                  | Their Folds,                   | — 5               | —                  | —                 |
|                        |                                  | Space for Roses,               | — 8               | Vessel.            | —                 |
|                        |                                  | Orle of the Vessel,            | — 1               | —                  | —                 |
|                        |                                  | Astragal,                      | — 3               | —                  | —                 |
|                        |                                  | Ove,                           | — 6               | —                  | —                 |
|                        |                                  | Reglet,                        | — 2               | —                  | —                 |
|                        |                                  | .....                          | .....             | .....              | .....             |
|                        |                                  | Goutiere,                      | — 6               | Abaque.            | 1 27              |
|                        |                                  | Reglet,                        | — 1               | 1                  | 28                |
|                        |                                  | Ove,                           | — 3               | 2                  | —                 |
| EN-                    | Architrave 1 <sup>m</sup> . 10'. | Bande 1.                       | — 11              | —                  | 26                |
|                        |                                  | Talon,                         | — 2 $\frac{3}{4}$ | —                  | —                 |
|                        |                                  | Bande 2.                       | — 15              | —                  | 28 $\frac{5}{12}$ |
|                        |                                  | Cord or Filet,                 | — 1 $\frac{1}{4}$ | —                  | 29 $\frac{1}{4}$  |
|                        |                                  | Talon,                         | — 3 $\frac{3}{4}$ | —                  | —                 |
|                        |                                  | Cavett,                        | — 4 $\frac{1}{8}$ | 1 inf.             | 2 $\frac{1}{2}$   |
|                        |                                  | Reglet,                        | — 2 $\frac{1}{8}$ | 1                  | 5 $\frac{3}{4}$   |



| Parts.                        | Members.     | Heights. |                   | Ecphoræ.                      |                                   |
|-------------------------------|--------------|----------|-------------------|-------------------------------|-----------------------------------|
|                               |              | m        | '                 | m                             | '                                 |
| TABLEMENT 4 <sup>m</sup> .    | Frize,       | 1        | † —               | below 26                      | † Its Ex-                         |
|                               |              |          |                   | 1 above                       | † tremities                       |
|                               | Reglet,      | —        | 1 $\frac{1}{4}$   | 1                             | are joyned                        |
|                               | Astragal,    | —        | 1 $\frac{1}{2}$   | 1                             | with an                           |
|                               | Talon,       | —        | 5                 | —                             | Arch.                             |
|                               | Reglet under | —        | 1                 | 1                             |                                   |
|                               | Modillions,  | —        | 1                 | 3 $\frac{1}{2}$               |                                   |
|                               | 1 Bande of   | —        | * 5               | 1 + 7 $\frac{1}{4}$           | * The first                       |
|                               | Modillions,  | —        | 1 $\frac{3}{4}$   | —                             | Bande of                          |
|                               | Talon,       | —        | 1 $\frac{3}{4}$   | —                             | Modillions                        |
|                               | 2 Bande of   | —        | 6 $\frac{1}{2}$   | 1                             | is continu-                       |
|                               | Modillions,  | —        | 6 $\frac{1}{2}$   | 8 $\frac{1}{4}$               | ed 14 $\frac{1}{2}$ .             |
| Cornice 1 <sup>m</sup> . 20'. | Cord,        | —        | 1                 | 8 $\frac{1}{2}$               | beyond the                        |
|                               | Ove,         | —        | 2 $\frac{1}{2}$   | 1 <i>sup.</i> 9 $\frac{3}{4}$ | Ecphora                           |
|                               | Goutiere,    | —        | x 9 $\frac{1}{2}$ | 2                             | set down                          |
|                               | Talon,       | —        | 3 $\frac{1}{2}$   | —                             | in the                            |
|                               | Reglet,      | —        | 1                 | 2                             | Table. So                         |
|                               | Doucine,     | —        | 8                 | —                             | the whole                         |
|                               | Reglet.      | —        | 2 $\frac{1}{2}$   | 2                             | Ecphora                           |
|                               |              |          |                   | 16                            | of the low-                       |
|                               |              |          |                   |                               | er Part of                        |
|                               |              |          |                   |                               | Modillions                        |
|                               |              |          |                   |                               | 1 <sup>m</sup> . 21 $\frac{3}{4}$ |

+ The Goutiere is hollowed below at 2'. Distance from its Extremity.

x In the first Bande of Modillions their Breadth is 9'  $\frac{1}{2}$ . and their Distance 23'. In the Second Bande their Breadth

## 248 *A short Introduction*

Breadth is  $12\frac{1}{2}$ . and their Distance or Metope 20'.

In the Cornice of the Entablement, the Talon above the first Bande serves as a Cymaife to its Mutules, as the Cord and Ove does to those of the Second Bande.

The Space lying betwixt the Tops of the Second Range of Leaves and the Reglet of the Abaque, is possessed by Two *Ionick* Volutes, the Center of each Volute is in the Line that limits the Projection of the Leaves.

### *The ROMAN or COMPOSITE ORDER according to SCAMOZZI.*

**T**HIS Author places this *Composite* Order in the Fourth Place, and the *Corinthian* in the last, as being by far the most delicate of all the Five Orders. He gives to the Column, including its Base and Capital,  $19^m. 15'$ . of which the Base has  $1^m.$  the Fust  $16^m. 5'$ . and the Capital  $2^m. 10'$ .

He allows  $\frac{4}{13}$  of the Height of the Column, or  $6^m.$  to the Pedestal, of which

which the Base has  $1^m. 15'$ . the Dy  $3^m. 22\frac{1}{2}'$ . and the Cornice  $22\frac{1}{2}'$ .

The Entablement has  $\frac{1}{3}$  the Height of the Column, or  $3^m. 27'$ . of which the Architrave has  $1^m. 9'$ . the Frize  $1^m. 1'$ . and the Cornice  $1^m. 17'$ .

Seeing now the Height of the Column is  $19^m. 15'$ . that of the Pedestal  $6^m$ . and that of the Entablement  $3^m. 27'$ . the whole Height of the Order shall be  $29^m. 12'$ .

| Parts.   |            | Members.    | Heights. |                 | Ecphoræ.      |                 |
|----------|------------|-------------|----------|-----------------|---------------|-----------------|
|          |            |             | m        | '               | m             | '               |
| PEDESTAL | $1^m. 15'$ | Socle,      | 1        | —               | 1             | 26              |
|          |            | Tore,       | —        | $3\frac{1}{2}$  | —             | —               |
|          |            | Filet,      | —        | 1               | 1             | 24              |
|          |            | Gueule inv. | —        | $5\frac{1}{2}$  | —             | —               |
|          |            | Astragal,   | —        | $1\frac{3}{4}$  | 1             | 16              |
|          |            | Filet,      | —        | $1\frac{1}{2}$  | 1             | 16              |
|          |            | Talon inv.  | —        | $2\frac{3}{4}$  | —             | —               |
|          | Cornice    | Dy,         | 3        | $22\frac{1}{2}$ | 1             | 12              |
|          |            | Talon,      | —        | 4               | —             | —               |
|          |            | Reglet,     | —        | 1               | 1             | 16              |
|          |            | Astragal,   | —        | $1\frac{1}{2}$  | 1             | 17              |
|          |            | Ove,        | —        | * 5             | 1 <i>sup.</i> | $19\frac{1}{2}$ |
|          |            | Filet,      | —        | * 1             | 1             | 24              |
|          |            | Goutiere,   | —        | $4\frac{1}{2}$  | 1             | $25\frac{1}{4}$ |

\*\* In the Reglet below the Goutiere is an Arch joyning the Ove, and leaving on the Out-side a Bande of 2'.



250 *A Short Introduction*

| Parts.                        | Members.                        | Heights.              |       | Ecphora.         |                        |
|-------------------------------|---------------------------------|-----------------------|-------|------------------|------------------------|
|                               |                                 | m                     | '     | m                | '                      |
| COLUMN 19 <sup>m</sup> . 15'. | 22' $\frac{1}{2}$ .             | Talon,                | —     | 3 $\frac{1}{4}$  | —                      |
|                               |                                 | Reglet,               | —     | 2 $\frac{1}{4}$  | 1 28 $\frac{1}{2}$     |
|                               | Base 1 <sup>m</sup> .           | Plinthe,              | —     | 9 $\frac{1}{2}$  | 1 12                   |
|                               |                                 | Tore,                 | —     | 7 $\frac{1}{4}$  | —                      |
|                               |                                 | Astragal,             | —     | 2 $\frac{1}{3}$  | 1 9                    |
|                               |                                 | Filet,                | —     | 1                | 1 8                    |
|                               |                                 | Scotie,               | —     | 3 $\frac{1}{2}$  | —                      |
|                               |                                 | Filet,                | —     | 1 $\frac{1}{2}$  | 1 6                    |
|                               |                                 | Tore,                 | —     | 4 $\frac{3}{4}$  | 1 8                    |
|                               | b Fust 16 <sup>m</sup> . 5'.    | Astragal,             | —     | 2 $\frac{1}{2}$  | 1 6                    |
|                               |                                 | Orle with Chanfrain,  | —     | 1 $\frac{1}{4}$  | 1 4 $\frac{1}{2}$      |
|                               |                                 | Trunk,                | 15    | 26 $\frac{1}{4}$ | above 26 $\frac{1}{4}$ |
|                               |                                 | Orle above with Chan. | —     | 2                | — 29 $\frac{1}{4}$     |
|                               |                                 | Astragal,             | —     | 3                | 1 $\frac{1}{4}$        |
|                               | c Capital 2 <sup>m</sup> . 10'. | Leaves 1.             | —     | 15               | —                      |
|                               |                                 | Their Folds,          | —     | 5                | —                      |
|                               |                                 | Great leaves,         | —     | 15               | —                      |
|                               |                                 | Their Folds,          | —     | 5                | —                      |
|                               |                                 | Space for Roses,      | —     | 7 $\frac{1}{2}$  | —                      |
|                               |                                 | Orle,                 | —     | 1 $\frac{1}{2}$  | —                      |
|                               |                                 | Astragal,             | —     | 3                | —                      |
|                               |                                 | Ove,                  | —     | 6                | —                      |
|                               |                                 | Space,                | —     | 2                | —                      |
|                               |                                 | .....                 | ..... | .....            | .....                  |

Parts,

| Parts.                           |                                 | Members.              | Heights. |                   | Ecphoræ.               |                  |
|----------------------------------|---------------------------------|-----------------------|----------|-------------------|------------------------|------------------|
|                                  |                                 |                       | m        | '                 | m                      | '                |
| ENTABLEMENT 3 <sup>m</sup> . 27. | Architrave 1 <sup>m</sup> . 9'. | Plinthe,              | —        | 5 $\frac{1}{4}$   | <sup>d</sup> Abaque, — | —                |
|                                  |                                 | Filet,                | —        | 1 $\frac{3}{4}$   | —                      | —                |
|                                  |                                 | Ove,                  | —        | e 3 $\frac{1}{2}$ | 2                      | —                |
|                                  |                                 | Bande 1.              | —        | 6 $\frac{1}{2}$   | —                      | 26 $\frac{1}{4}$ |
|                                  |                                 | Astragal,             | —        | 1 $\frac{1}{2}$   | —                      | 27 $\frac{3}{4}$ |
|                                  |                                 | Bande 2.              | —        | 8 $\frac{2}{3}$   | —                      | 27 $\frac{3}{4}$ |
|                                  |                                 | Talon,                | —        | 2 $\frac{1}{4}$   | —                      | —                |
|                                  |                                 | Bande 3.              | —        | 11 $\frac{1}{2}$  | —                      | 29 $\frac{1}{4}$ |
|                                  |                                 | Astragal,             | —        | 1 $\frac{3}{4}$   | 1                      | 2                |
|                                  | Cornice                         | Talon,                | —        | 4 $\frac{1}{2}$   | —                      | —                |
|                                  |                                 | Reglet,               | —        | 2 $\frac{1}{2}$   | 1                      | 1 $\frac{3}{4}$  |
|                                  |                                 | Frize,                | 1        | 1                 | —                      | 26 $\frac{1}{4}$ |
|                                  |                                 | Talon,                | —        | 4                 | —                      | —                |
|                                  |                                 | Filet,                | —        | 1                 | 1                      | 28 $\frac{1}{2}$ |
|                                  |                                 | Goutiere,             | —        | 5 $\frac{1}{4}$   | 1                      | 3                |
|                                  |                                 | Filet,                | —        | 1                 | 1                      | 3 $\frac{1}{2}$  |
|                                  |                                 | Ove,                  | —        | 4 $\frac{1}{2}$   | 1 <sup>sup.</sup>      | 7 $\frac{1}{2}$  |
|                                  |                                 | 1 Bande of Modillons, | —        | f 3               | 1 f                    | 25 $\frac{1}{4}$ |
|                                  |                                 | Talon,                | —        | 1 $\frac{1}{2}$   | 1                      | 26               |
|                                  |                                 | 2 Bande of Modillons, | —        | 4 $\frac{1}{2}$   | 1                      | 26               |
|                                  |                                 | Cord,                 | —        | 1                 | —                      | —                |
|                                  |                                 | Ove, Cy-              | —        | z 2               | 1                      | 27 $\frac{1}{3}$ |
|                                  |                                 | maise of Modillons,   | —        | —                 | —                      | —                |

| Parts.                             | Members   | Heights.                       | Ecphoræ.                       |
|------------------------------------|-----------|--------------------------------|--------------------------------|
|                                    |           | m                              | m                              |
| 1 <sup>m</sup> . 17 <sup>l</sup> . | Goutiere, | — <sup>h</sup> 6 $\frac{1}{2}$ | 2 <sup>h</sup> 4 $\frac{1}{4}$ |
|                                    | Filet,    | — 1                            | 2 4 $\frac{1}{2}$              |
|                                    | Talon,    | — 3                            | — —                            |
|                                    | Filet,    | — $\frac{1}{4}$                | 2 8 $\frac{3}{4}$              |
|                                    | Doucine,  | — 6                            | — —                            |
|                                    | Reglet.   | — 2                            | 2 15 $\frac{1}{4}$             |

<sup>a</sup> In the Fust the Canals are Twenty four in Number, and semicircular, the Listel is  $\frac{1}{3}$  the Canals Breadth.

<sup>b</sup> In the Abaque is a Flower 15'. large, from the Sides of that Flower the Volutes take their Arise, and ascending touch the Filet of the Abaque, then descend so as to touch the Second Leaves with the lower Part of their Curvature.

<sup>c</sup> The Volutes are *Ionick*.

<sup>d</sup> The Profil of the Projection of the Parts of the Abaque is the same as in the *Corinthian Order*.

<sup>e</sup> On the higher Part of the Ove, and from the Axis, set off 1<sup>m</sup>. 22'  $\frac{1}{2}$ . a Line drawn from that Division to the Extremity of the Astragal of the Capital, limits the Projections of the Leaves,

<sup>f</sup> The



<sup>ff</sup> The first Bande of Modillions is continued  $17\frac{3}{4}$ . beyond the Ecphora, in the Table; of these  $17\frac{3}{4}$ . there are  $13\frac{3}{4}$ . cut so as to leave a Reglet of 1'. Height, leaving in the Extremity a Bande of 4'.

<sup>g</sup> The Talon above the first Bande of Modillions serves as a Cymaïse to that Bande, as the Cord and Ove above the Second Bande of Modillions do to that Bande.

The Breadth of the Modillon in the first Bande is 9'. their Metope 24'. In the Second Bande the Breadth of the Modillon is 12'. and their Metope 21'.

<sup>hh</sup> The Goutiere above the Ove has in its lower Part a circular Hollow of 3'. that joyns the Ove, and leaves on its Extremity a Bande of 3'.

### PROB. I.

*TO reduce the Ratio's of Modules and Minutes to the like Ratio's of Feet and Inches.*

Say, As the Radius of the lower Part of the Fust, *i. e.* 1<sup>m</sup>. or 30'. is  
to

## 254 *A Short Introduction*

to the Height or Ecphora of the given Member in Modules and Minutes, so is the Number of Feet and Inches in the Radius of the lower Part of the Fust to a Fourth Proportional, which gives the Height or Ecphora of the given Member in Feet and Inches.

Plate XIX.  
FIG. 1.

### PROB. II.

*TO make a Scale of Modules and Minutes.*

Divide a given Line  $AB$  into any Number of equal Parts, divide one of its extreme Parts  $CB$  into Three equal Parts, at the Points  $10$  and  $20$ . draw  $BE \perp AB$ , and divide it into Ten equal Parts, draw  $ED \parallel$  and  $= AB$ , joyn  $AD$ , through the Divisions of  $BE$  draw Lines parallel to  $AB$ , divide  $DE$  as  $AB$  is divided, at the Points  $o, n, m, F, G, K$ , draw  $CG, 10K, 20E$ , and it is plain, that making  $CB$  one Module, the Divisions of  $CG, 10K, 20E$ , shall be Minutes.

Plate XIX.  
FIG. 2.

### PROB. III.

*TO lay down on Paper any of the Five Orders.*

Draw

Draw  $AB$ ,  $AC$  making a right Angle at  $A$ , draw the faint Line  $DE \perp AB$ , representing the Axis of the Column continued; on each Side of the Point  $D$ , and, on the Line  $AB$ , set off the *Ecphoræ*, these shall be limited in the Draught of the Column by Perpendiculars drawn to  $AB$ , through their proper Division. In the same Manner on  $AC$  set off the Heights of the several Members, and Perpendiculars to  $AC$ , drawn through these Divisions, shall limit their several Heights.

*N. B.* The Heights and *Ecphoræ* of the Members of the several Orders, in *Plates XVIII and XIX.* are taken from the Tables already laid down, and are measured on the Scale described in the preceeding Problem, and are so disposed, that beginning with the Socle of the Base of the Pedestal, and ascending, you have the several Parts and Members in the same Order and Proportion as they are in the Tables.

The Frontispiece is always placed directly above the Cornice of the Entablement; its Parts, as was before shewn, are, the Tympan, Cornice and Acroteres, the Members of its Cornice are the same with those of the Cornice of the Entablement, except that the last is generally either an Ove or a Doucine.

The



## 256 *A short Introduction*

The Acroteres consist of a Dy and a Cymaife, and are nothing else but Pedestals for Statues.

The Tympan may either be left void, or supplied with Ornaments of Sculpture.

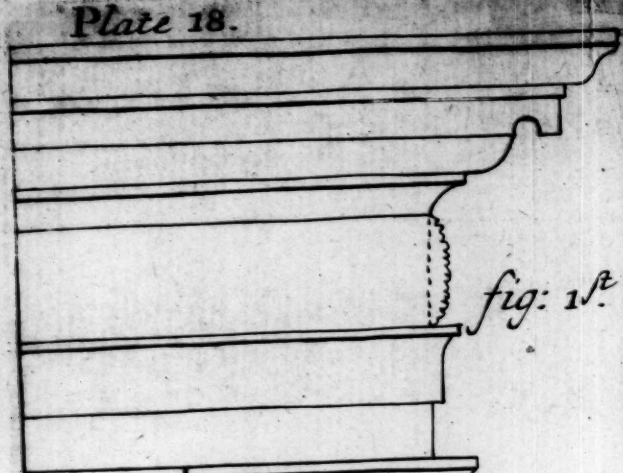
Plate XIX.  
FIG. 3.

The whole Structure of the Frontispiece consists in finding its Height, which is done thus.

Divide the whole Length of the Goutiere  $qt$  into Nine equal Parts, according to *Vitruvius*, or, according to others, into Seven, then, at  $p$ , the middle Point, raise  $PF \perp qt$ , and equal to one of these Parts, that  $PF$  gives the Height of the Frontispiece.

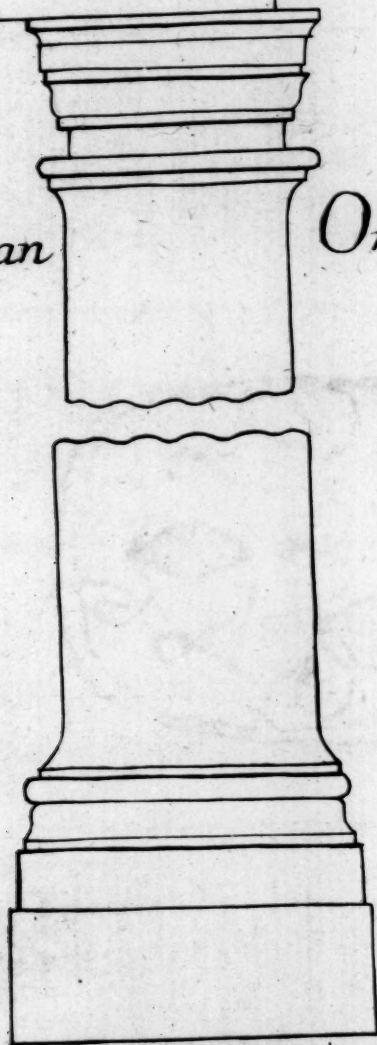
*F I N I S.*





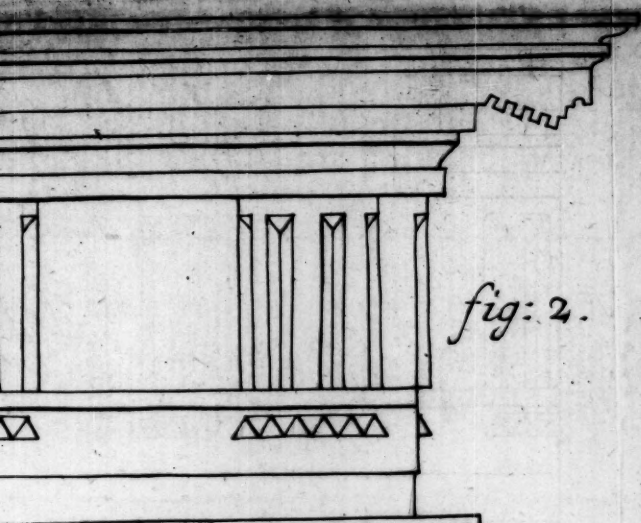
Tuscan

Order



Palladio.

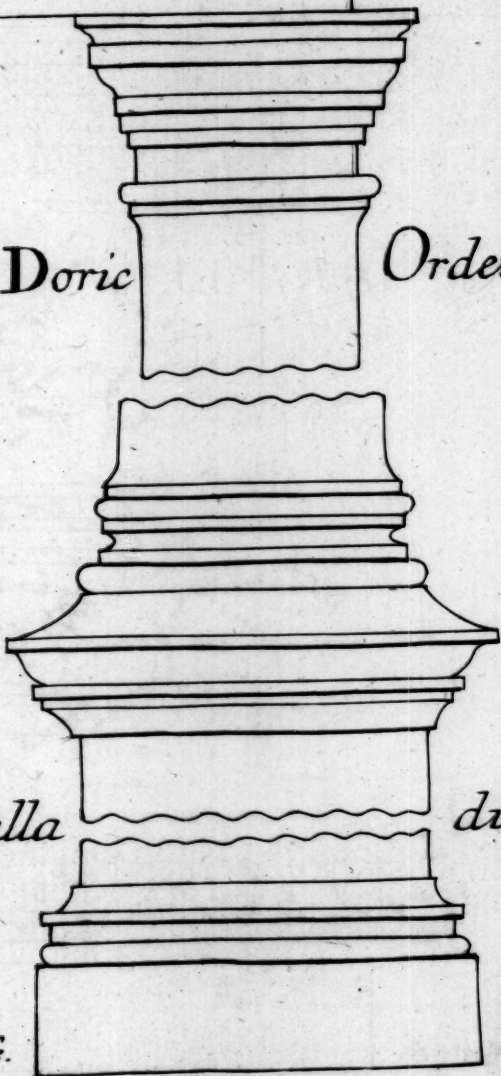
Ro: Mybne sc.



*fig: 2.*

*Doric*

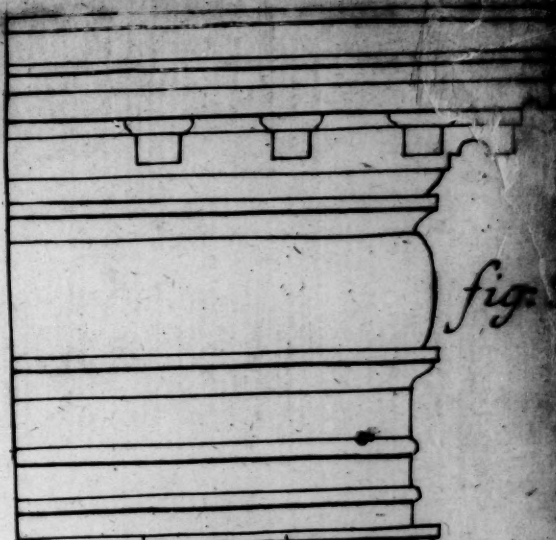
*Order*



*Palla*

*dio.*

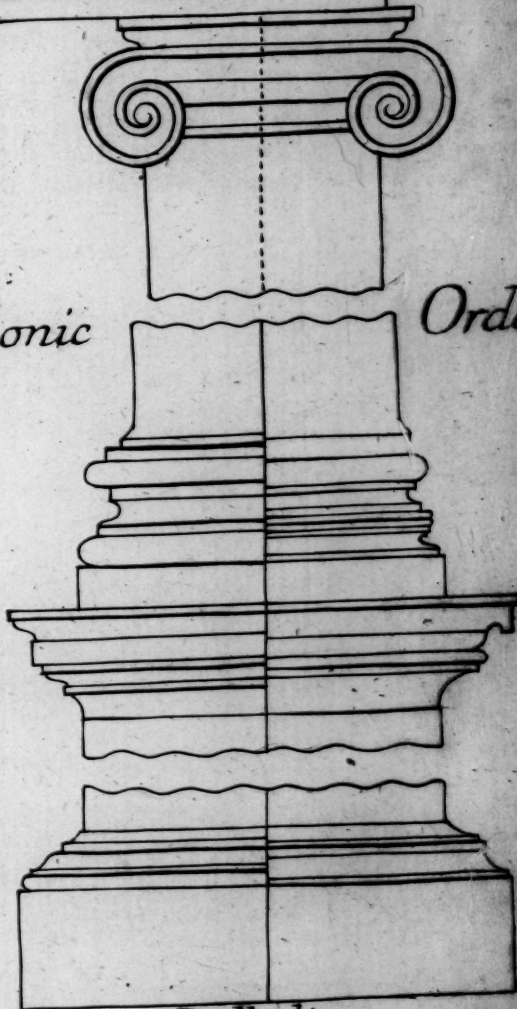
*the sc.*



*fig: 3.*

*Ionic*

*Order*



*Palladio.*



Corinthian

C

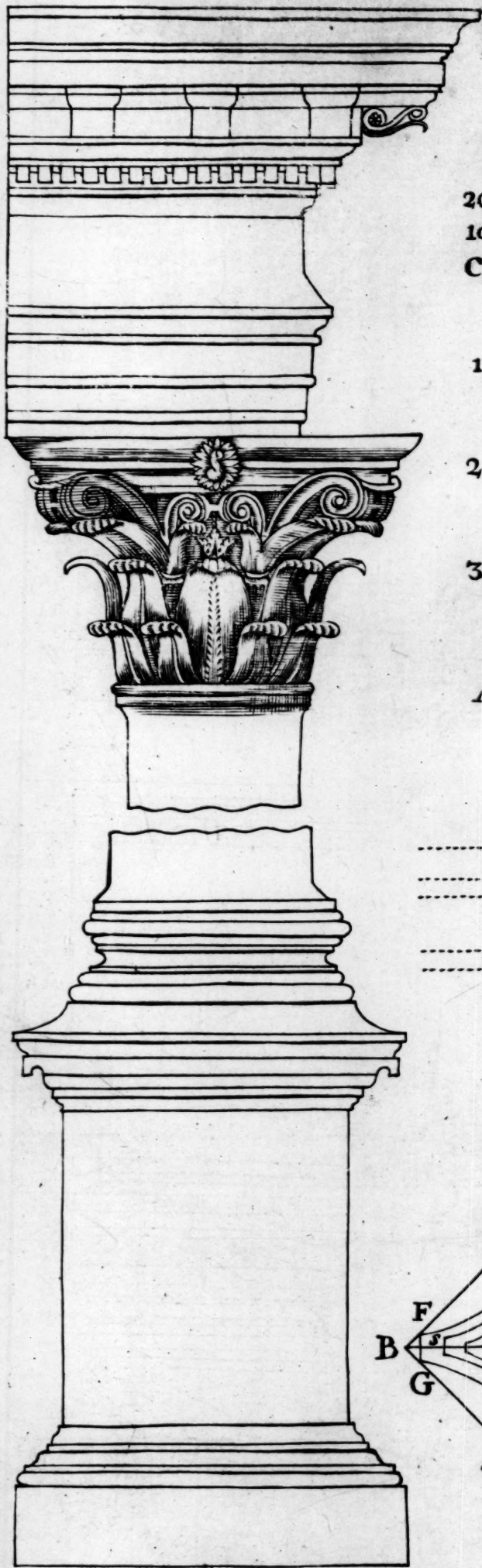
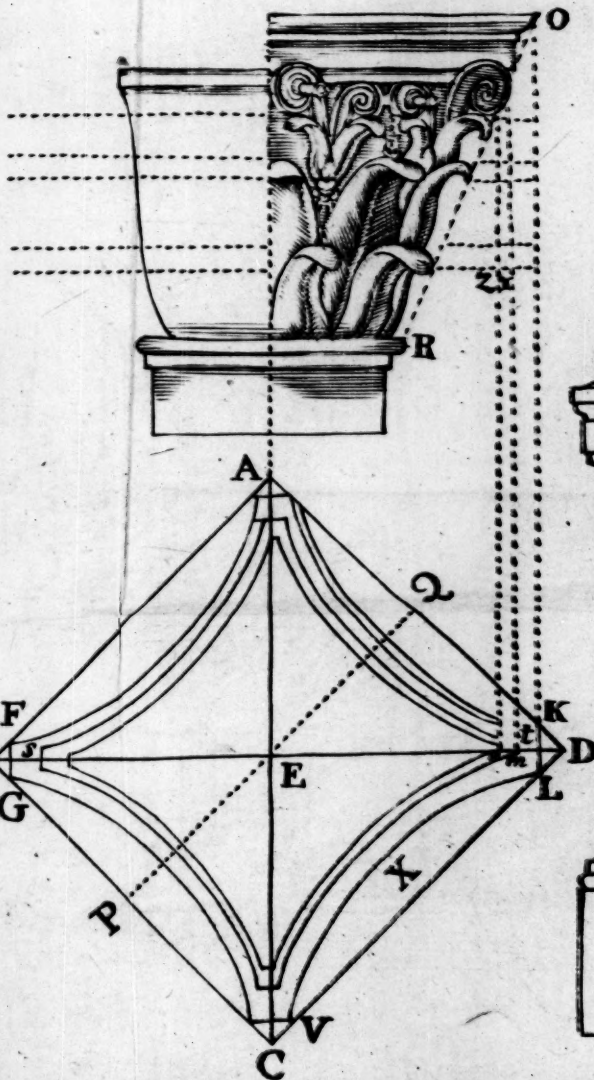
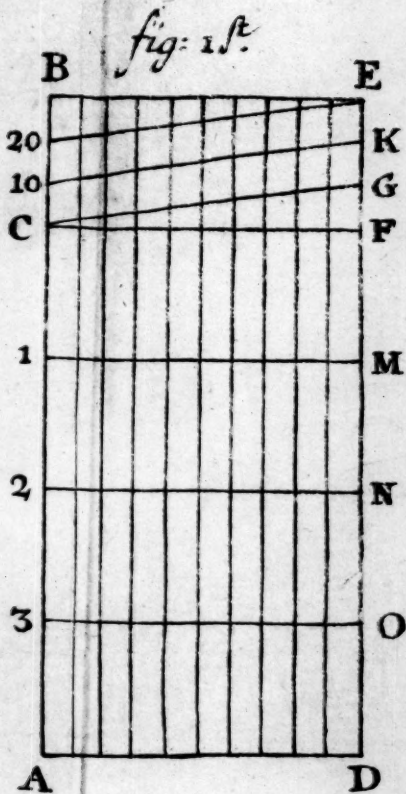


fig.  
2

A

D

Palladio



Palladio